Handbook of the
5th World Congress on Paraconsistency

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5th WCP
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Indian Statistical Institute
Kolkata — India

Edited by
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- ✦ John Woods, Department of Philosophy, University of British Columbia, Canada

2 – Fifth World Congress on Paraconsistency

2.1 What is Paraconsistent Logic?

Paraconsistent logic is a field of research based on the distinction between contradiction and triviality.

The expression was coined by the Peruvian philosopher Miró Quesada as an answer to Newton da Costa looking for a good name for the systems he was working on.

There are many different systems of paraconsistent logic based on different techniques.

Paraconsistent logic is connected with deep philosophical issues regarding the nature of negation and reality and it has a lot of applications ranging from geometry to washing machines, through medicine, law and music.

2.2 Aim of the event

This is the 5th world congress on paraconsistent logic (WCP5), gathering top researchers from all over the world, after

— WCP1, Ghent, Belgium, 1997;

— WCP2, Juquehy, Brazil, 2000;

— WCP3, Toulouse, France, 2003;

— WCP4, Melbourne, Australia, 2008.

All aspects of paraconsistency are under examination: studies of various systems of paraconsistent logic, general tools and frameworks for these systems, philosophical discussion and historical investigations as well as challenging applications.

The WCP5 is emphasizing an interdisciplinary perspective ranging from mathematics to arts, through computer science, quantum physics, artificial intelligence, philosophy and linguistics.

2.3 Publications

The following books have been released:

— WCP1 = *Frontiers of Paraconsistent Logics*, edited by D. Batens, C. Mortensen and J.-P. van Bendegem, Research Studies Press, Baldock, 2000.

— WCP2 = *Paraconsistency: The Logical Way to the Inconsistent*, edited by W.A. Carnielli, M.E. Coniglio and I.M.L. D'Ottaviano, Marcel Dekker, New York, 2002.

— WCP3 = *Handbook of Paraconsistency*, edited by J.-Y. Beziau, W.A. Carnielli and D.M. Gabbay, College Publications, London, 2007.

— WCP4 = *P*araconsistency: Logic and Applications, edited by K. Tanaka, F. Berto, F. Paoli and E. Mares, Springer, Dordrecht, 2013.

The following book will be published:

— WCP5 = *New Directions in Paraconsistent Logic*, edited by J.-Y. Beziau, M.Kr. Chakraborty and S. Dutta, Springer, New Delhi, 2014.

Participants should submit full versions of their papers by May 15, 2014 to wcp5@paraconsistency.org. People not taking part of the event are also welcome to submit a paper.

2.4 Call for papers

To submit a contribution send a one page abstract to wcp5@paraconsistency.org. All talks related to paraconsistent logic are welcome, in particular those falling into the categories below. Artistic works related to paraconsistency are also welcome: pictures, paintings, music, movies; send your files to paraconsistency.wcp5@gmail.com. The deadline for submission is May 15, 2014.

Systems of Paraconsistent Logic:

- ✦ imaginary logic,
- ✦ discussive logic,
- ✦ C-systems,
- ✦ logics of strong negation,
- ✦ many-valued paraconsistent logics,
- ✦ paraconsistent fuzzy logic,
- ✦ classical paraconsistent logics,
- ✦ dual-intuitionistic logics,
- ✦ paraconsistent quantum logics,

- modal paraconsistent logics,
- non-monotonic paraconsistent logics,
- paranormal logics,
- deontic paraconsistent logic,
- logics of formal inconsistency,
- paraconsistent set theory,
- paraconsistent relevant logics.

General Theory and Tools:

- definition of paraconsistency,
- replacement theorem and paraconsistency,
- cut-elimination and paraconsistency,
- truth functionality and paraconsistency,
- incompleteness and paraconsistency,
- category theory, algebra and paraconsistency.

Philosophy:

- the nature of negation,
- the principle of non-contradiction,
- fallacies,
- contradiction and reality,
- dialetheism,
- contradiction and language,
- contradiction and god,
- opposition.

Applications:

- expert systems,
- psychoanalysis,
- medicine,
- duality wave-particle,
- geometry,
- arithmetics,
- linguistics and semiotics,
- argumentation and discourses,
- law and justice,
- moral dilemmas,
- resolution of paradoxes.

History:

- Aristotle and non-contradiction,
- Ex-falso sequitur quod libet,

- Hegel's dialectic,
- Vasiliev and non-Aristotelian logic,
- Łukasiewicz and non-contradiction,
- Contradiction and Tao,
- Contradiction in Indian thought.

2.5 Contest — Picturing Contradiction

The contest *Picturing Contradiction* is open to anyone (no obligation to be present during the event). Send an image picturing contradiction by February 1st, 2014 to paraconsistency.wcp5@gmail.com, with about 15 lines, explaining why it is a good picture of contradiction.

The best pictures will be selected and posted during the event.

The winner will be awarded the *Handbook of Paraconsistency* and his picture will be printed in the WCP5 book, to be published by Springer.

Members of the jury for the contest PICTURING CONTRADICTION are Mihir Kumar Chakraborty, Catherine Chantilly and Chris Mortensen.

3 – Tutorials

3.1 Adaptive Logics

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The adaptive logics program aims at elaborating a unified framework for characterizing defeasible reasoning forms, including dynamic proofs that may serve as an explication of actual reasoning. The tutorial will consist of two parts.

The first part will be devoted to the standard format (SF) of adaptive logics. Properties of defeasible reasoning forms will be reviewed and the SF will be presented as a means to provide those reasoning forms with a dynamic proof theory, a selection semantics, and most of the metatheory. Some recent insights led to a nice and simple version of the SF. This version allows for the direct incorporation of many defeasible reasoning forms and for the characterization (through a very conservative translation function) of many others.

The running example will concern logics that handle inconsistency in a defeasible way, viz. interpret inconsistent premise sets as normally (consistently) as possible. Other adaptive logics will be brought in to illustrate the generality of the program.

The second part concerns the large variety of adaptive logics that may be invoked to handle inconsistency. As the nature of these logics is methodological rather than deductive, their multiplicity is a highly desirable property. We shall consider variants of the (three) elements of the SF. This will involve different logics that may function as a deductive starting point (the so called lower limit logic), different sets of abnormalities, and different adaptive strategies. Several unexpected features will turn up. The lower limit logic will not always be paraconsistent. Minimizing several kinds of abnormalities results in impressively different approaches. Varying strategies opens interesting perspectives on the nature of the choices to be made. Some examples of combined adaptive logics will exemplify further unexpected perspectives: they overcome weaknesses that seem to result from the choice of a lower limit logic.

The second part covers a diversity of technicalities but also opens up philosophical perspectives on the justification of certain logical and methodological choices. Attention will be paid to application contexts and to the interplay between, on the one hand, logical and methodological considerations and, on the other hand, considerations that involve metaphysical and other substantive choices.

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3.2 General Theory of Paraconsistent Negation

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The aim of this tutorial is to explain how we can define a paraconsistent negation and what kind of paraconsistent systems can be constructed, There is no prerequisites for this tutorial, just an acquaintance with abstract thinking. It can be considered as an introduction to paraconsistent logic.

We will start with a presentation of the various formulations of paraconsistent negation and their interrelations. We will discuss the interplay between philosophical expressions of the principle of non-contradiction and their different formalizations. Terminologies like inconsistency, trivialization and contradiction will be analyzed. We will emphasize the distinction between positive and negatives criteria for the definition of a paraconsistent negation.

We will then present the different properties of negation and show how they can be articulated: double negation, reduction to the absurd, de Morgan laws, contraposition, ex-falso sequitur quodlibet, principle of non-contradiction, replacement theorem, maximality. On this basis we will explain how the different systems of paraconsistent logic can be classified, if it makes sense or not to say that such or such system is paraconsistent, for example the minimal logic of Johansson.

We will discuss the different tools that can be used to develop paraconsistent negations: sequent systems, logical matrices, bivaluations, possible world semantics and will study the relations between paraconsistent logic and other non-classical logics: classical logic, intuitionistic logic, modal logic, many-valued logic, fuzzy logic, non-monotonic logic, relevant logic.

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3.3 On the Philosophy and Mathematics of the Logics of Formal Inconsistency

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This tutorial is divided into two parts. The first part deals with mathematical and logical aspects of the logics of formal inconsistency (**LFI**s), and will be devoted to a gentle introduction to the main concepts and methods of the **LFI**s, which are logical systems that treat consistency and inconsistency as mathematical objects. Such logics allow the internalization of the notions of consistency and inconsistency at the object-language level, resulting in very expressive logical systems whose fundamental feature is the ability of recovering all consistent reasoning, while still allowing to reasoning under contradictions.

Several families of well-known paraconsistent logics can be expressed under the viewpoint of the **LFI**s. A subclass of **LFI**s where consistency can be expressed as a unary connective defines the so-called **C**-systems. Further, the **dC**-systems are introduced as the **C**-systems in which the consistency connective is explicitly definable in terms of other usual connectives. Particular cases of **dC**-systems are the famous da Costa's logics $\mathbf{C_n}$, $1 \leq n < \omega$, Jaśkowski's logic **D2**, and most of the normal modal logics under convenient formulation.

The logic **mbC**, a fundamental example of **LFI**, will be treated in detail, and we will show how to introduce a large family of logics by controlling the propagation of consistency, clarifying a procedure that allows one to define tailor-suited **LFI**s. A brief explanation on how to build new and sophisticated paraconsistent set theories by talking about consistent and inconsistent sets (as well as consistent and inconsistent sentences) will also be considered. Other topics like completeness of **LFI**s under the possible-translations semantics, uncharacterizability of most **LFI**s by finite matrices, and some perspectives of applications of **LFI**s will also be sketched.

The second part deals with some philosophical aspects of the logics of formal inconsistency. The following topics will be covered: on the nature of logic: ontological, epistemological and linguistic aspects of logic; paraconsistency and logical realism; paraconsistency from the ontological and the epistemological viewpoints; **LFI**s and the descriptive character of logic; consistency as a primitive concept; expressing consistency in the object language; how to understand paraconsistent negations, and **LFI**s as extensions of classical logic.

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4 – Round Tables

4.1 Paraconsistent Logic and Reasoning

Coordinator:

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Participants:

Can Başkent, Diderik Batens, Jean-Yves Beziau and Zach Weber

Logic can be both considered as reasoning and the study of reasoning. A logic can be alternatively seen as a way of reasoning, a description of reasoning, and a tool for reasoning. A paraconsistent logic is defined as a logic in which there is a negation such that from a proposition and its negation it is not possible to derive everything, but such that the other laws of usual logic would be preserved as much as possible. In this round table we discuss the following questions:

— Is paraconsistent logic a good description of the way we actually reason?
— Does paraconsistent logic provide helps to understand the subtleties and complexities of reasoning?
— Does paraconsistent logic suggest (or describe) a new way of reasoning that can be usefully developed by human beings and machines?

4.2 Brain, Contradictions and Computability

Coordinator:

Sisir Roy, Indian Statistical Institute, Kolkata, India

Participants from India:

Palash Sarkar, B.P. Sinha, Guruprasad Kar and Joby Joseph

Participants from outside India:

Graham Priest, Christian de Ronde and Walter Carnielli

Use of logic in modeling brain functions is an enduring theme. McCulloh and Pitts (1943) demonstrated that neuronal networks carry out logical operations. Their work is based on the proposition that every neural activity is a computation and every mental activity is explained by some neural computation. The novelty of McCulloch and

Pitts's paper lies in the fact that they employed logic and the mathematical notion of computation — introduced by Alan Turing (1936–37) — in terms of what came to be known as Turing Machines — to explain how neural mechanisms might realize mental functions. Some recent works claimed that cognitive processes could be built up using the networks of neural elements which can learn basic operations of logic. However, convincing evidence of implementation of logical operations by brain is still lacking.

The developments of Bayesian approach to brain functions clearly indicate that probabilistic models are distinctly better than those based on formal logic to understand the response properties of neurons. The response of neurons at various levels of sensory hierarchy may be described better by probability than by formal logic. Recently, through several experiments with human subjects, violation of traditional Bayesian probability theory is clearly revealed in plenty of cases. Literature survey clearly suggests that classical probability theory fails to model human cognition beyond a certain limit. The empirical findings (categorized in six group of empirical evidences) in human judgment related to order/context effects, violations of the law of total probability and failures of compositionality suggest that a reformulation of Hierarchical Bayesian theory of inference under this set-up or a more general probabilistic framework based approach would be more plausible than a Bayesian model or the standard probability theory. A generalized version of probability theory borrowing ideas from quantum mechanics may be a plausible approach. Quantum theory allows a person to be in an indefinite state (superposition state) at each moment of time. A person may be in an indefinite state that allows all of these states to have potential (probability amplitude) for being expressed at each moment (Heisenberg, 1958). Thus a superposition state seems to provide a better representation of the conflict, ambiguity or uncertainty that a person experiences at each moment. Da Costa et al (2013) proposed a paraconsistent approach to quantum superpositions which attempts to account for the contradictory properties present in general within quantum superpositions. The epistemological issues associated to these types of contradictions in superposition states have been extensively discussed in Buddhist Logic many centuries before.

It is worth mentioning that neuroscientists (Llinas et al) raise a more fundamental issue whether the very concept of computability is applicable to understand functioning of brain. Their arguments essentially state that the laws of physics may be able to explain the functioning of brain and its "subjective experiences". In physics, to describe the falling of apple or the movements of the planets around closed orbit, we need not include the proposition that the apple or the planets compute their velocities or time elapsed during their pathways. We use a kind of mathematical language to describe the phenomena. The continuing challenge remains — "brain computes contradictions" is at all a meaningful statement or not.

5 – Talks of Invited Speakers

5.1 Some adaptive contributions to Logics of Formal Inconsistency

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The main aim of the paper is to present several theorems the proofs of which rely on theorems from [1] and [2]. Most of the new theorems concern the extension of LFI-theories by consistency statements (statements of the form $\circ A$ in which $\circ$ is a consistency operator).

Several facts about LFIs will be reviewed, some pertaining to the different consistency operators that may turn a paraconsistent logic into a LFI. Examples of LFIs will be considered, including some less usual ones.

Turning to the main aim, an essential question is which consistency statements should or may be added to inconsistent theories — these theories may but need not contain consistency statements themselves. Not adding enough will lead to too weak a theory, adding too many (or the wrong ones) will cause triviality. It is at this point that insights from the metatheory of adaptive logics will be invoked to delineate the set of maximal sets of consistency statements that an inconsistent theory will tolerate. Note that this task can be carried out by invoking logical considerations, while choosing one of these sets requires non-logical arguments.

A different topic is the articulation of adaptive LFIs. No general procedure seems to transform all LFIs into adaptive LFIs, but large sets of LFIs may be handled by the same means. It is instructive to compare adaptive LFIs with inconsistency-adaptive logics in which no consistency operator is definable.

The central feature of adaptive LFIs is that their consequence sets contain consistency statements that are not derivable from the premises by the original LFI. The Minimal Abnormality strategy delivers the consistency statements (and their consequences) that are members of every maximal set of consistency statements that the premise set tolerates. The Normal Selections strategy delivers the set of those maximal sets. A choice for a specific maximal set requires a non-logical rationale.

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5.2 How we live without detachment and enjoy both transparent truth and classical mathematics

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I discuss the viability of fully embracing that there are no detachable conditionals at all. I explain away the appearance of detachment via extra-logical resources. I then discuss how we can nonetheless enjoy fully classical theories of many things — particularly in mathematics.

5.3 Paraconsistent Logic and contradictory viewpoints

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We start by recalling the definition of contradiction from the perspective of the square of opposition, emphasizing that it comes together with two other notions of oppositions, contrariety and subcontrariety. We then introduce the notion of paraconsistent negation as a non-explosive negation; we explain the connection with subcontrariety and why it is better not to talk of contradiction in case of paraconsistent negation. We then explain that we can interpret the paradoxical duality wave/particle either as a subcontrariety in reality or as different contradictory viewpoints. We go on developing a logic based on a relational semantics with bivaluations conceived are viewpoints and in which we can define a paraconsistent negation articulating the oppositions between viewpoints.

After proving some basic results about this logic, we show the connection with modalities: we are in fact dealing with a reconstruction of S5 from a paraconsistent perspective and our paraconsistent negation is the classical negation of necessity. We finish by presenting a hexagon of opposition describing the relations between this negation, the negated proposition, classical negation and necessity.

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5.4 Symmetrical Preservation Relations and Cognitive Commitments

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Cognitive commitments and logical consequence are closely linked. One common way to express the link is to say that a commitment to the premises of a valid argument implies a commitment to its conclusion, and more generally, commitment to the closure of those premises under $\vDash$. This account of cognitive commitments fits easily with the usual single-conclusion presentation of consequence relations. By contrast, using multiple-conclusion relations to characterize our cognitive commitments can seem obscure — Restall (2005) proposes a straightforward reading of multiple conclusion consequence relations in terms of assertion and denial: $\Gamma \vDash \Delta$ holds iff the assertion of all members of Γ is *incompatible* with the denial of all members of Δ. But this incompatibility-based approach leaves open the question of what and how a multiple conclusion $\vDash$ tells us about the assertion commitments that follow from assertion of some premises (as well as the denial commitments that follow from denial of some set of sentences). The standard single-conclusion (or, for denial, the dual, single-premise) closure-accounts are still available. But this is disappointing. For many, it makes adopting a multiple-conclusion consequence relation more trouble than it's worth, at least when the aim is to specify the contents of our commitments.

Here I aim to generalize the preservationist approach to logic and to identify properties shared by a wide range of logics. A symmetrical consequence relation connects premises and conclusions of the same type: thus both single-premise/ single-conclusion and multiple-premise/ multiple-conclusion logics would count as symmetrical consequence relations. The family of consequence relations examined in this paper is characterized by a very general 1/0 semantics, in the sense of Scott(1974): in the single premise and conclusion case, $A \vDash B$ iff every *allowed valuation* V such that $V(A) = 1$ is also such that $V(B) = 1$, while in the multiple premise and conclusion case, $\Gamma \vDash \Delta$ iff every *allowed valuation* V such that $\forall\gamma \in \Gamma, V(\gamma) = 1$ is also such that $\exists\delta \in \Delta, V(\delta) = 1$. Scott showed that a multiple-conclusion consequence relation meets this condition iff it is monotonic, reflexive and transitive. Much the same goes for single-conclusion logics (Payette and Schotch, 2013).

This approach is *preservationist* in the sense that a consequence relation preserving a property of sentences from premises to conclusions throughout a range of cases can be expressed by a set of allowed 1/0 assignments to the sentences of a language, where each assignment distinguishes the sentences having the property to be preserved from those lacking it. Naturally, such logics can be read as logics of assertion and denial, as Restall(2005) proposes.

But thinking in these terms also points towards a general view of cognitive commitment, based on a dual relation between the family of premise sets from which a given conclusion or conclusion set follows, and the family of conclusion sets which follow from a given premise or premise set. In the singleton case, of course, the union of the family of conclusions that follow from a given premise A is the logical closure of the premise: $Cl\langle A, \vDash\rangle$, including all the sentences that receive the value 1 whenever A does. Conversely, the set of all premises from which a given conclusion sentence B follows is right-to-left logical closure of B, i.e. the set of all sentences that receive the value 0 whenever B does. But something more interesting emerges when we extend this approach to multiple premises and conclusions. Here 'aggregating' the conclusion sets that follow from a given premise set is done, not by gathering them together into a set, but by constructing a family of sets of sentences *dual* to the family of conclusion sets, the *least transverses* of the conclusion sets. These are defined as the minimal sets that intersect every conclusion set. If the premise set is satisfiable, these are the complete *satisfiable* sets including the premise set, i.e. the extensions of the premise set corresponding to allowed 1/0 valuations. Symmetrically, if a conclusion set is dissatisfiable, the least transverses of the premise sets it 'follows' from are the complete 'dissatisfiable' extensions of that conclusion set.

Here we find a helpful response to the opening question. Rather than think of our cognitive commitments as the logical closure of our explicitly stated commitments, we can think of them as an indeterminate or disjunctive commitment to the *maximal satisfiable/dissatisfiable extensions* of those explicit commitments.

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5.5 Paraconsistency and fuzziness

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Real-world reasoning is much more intricate than the classical account of logic is prepared for. It is commonly recognized that incomplete, uncertain or imprecise information, which often occurs in engineering applications, require a specific mathematical treatment. The idea of fuzzy sets, famously proposed by L. Zadeh, offers practical handling for such questions, but it does not add much to the foundational debate around vagueness and reasoning.

The so-called Mathematical Fuzzy Logic (MFL), to which P. Hájek dedicated his book [8], was thought as a foundational counterpart for the application-driven fuzzy sets, seen as a branch of applied mathematics. An important fuzzy logic named Basic Logic (BL) is introduced and treated in details in [8], generalizing previous attempts to found a fuzzy logic by means of Łukasiewicz, Gödel-Dummet and Product Logics. BL is the first deep and wide account of fuzziness as a logic based on the idea of continuous t-norms, of which each one of the above mentioned logics captures but a specific side (i.e, they are just particular continuous t-norms). The Monoidal t-Norm Logic (MTL) was introduced in [7] as a generalization of BL, intended to capture the semantics induced by left continuous t-norms and their residua.

But imprecision and incompleteness are not the only thorns in the side of traditional logic: another one is the question of formalizing reasoning under contradiction, given that a negation $\neg$ is around. The general domain of paraconsistency is devoted to the study of logic systems (the paraconsistent logics) with a negation operator $\neg$, such that not every contradictory set of premises $\{\varphi, \neg\varphi\}$ trivializes the system. Thus, any paraconsistent logic contains at least a contradictory but non-trivial theory.

Among the several systematic approaches to paraconsistency, the Logics of Formal Inconsistency (LFIs) introduced in [3] and more extensively treated in [2], are one of the most general and philosophically acceptable. The main characteristic of the LFIs is that they internalize the notions of consistency and inconsistency at the object language by means of specific connectives (primitive or not), what makes them very flexible and independent of exotic interpretations (see [1]). LFIs constitute a generalization of the well-known da Costa's C-systems, and at the same time generalize several other families of logic.

Although it seems just natural to combine the fuzzy and the paraconsistent paradigm in order to obtain a logic that would treat two subtleties of reasoning on a uniform basis, the task is not so easy. L.A. Zadeh in [12] surveys the torrid debate around the idea of fuzziness in its beginnings, as well as the confusion between fuzzy sets, fuzzy logics, many-valued logics and the debate whether fuzzy logics would be seen merely as logics of fuzzy concepts, or logics which would be themselves fuzzy. D. Hyde in [11], for instance, warns that (at least from some perspectives) a paraconsistent approach "remains a contender in accounting for vagueness".

D. Hyde and M. Colyvan argue in [10] that paraconsistent accounts of vagueness deserve further attention, specially when one takes into account that early formalizations of paraconsistent logics were treated as logics of vagueness. S. Halldén [9], where a three-valued logic to model vague predicates is proposed, is a clear case in this direction (his logic also happens to be paraconsistent, indeed an LFI, though this was clearly not his intention). Although their position is a bit too timid, as they do not see paraconsistency further than two or three somewhat worn-out alternatives, the objections in [10] against the negative claim that a paraconsistency account of vagueness would not deserve further consideration serve as a good starting point (see [1] for a comparison).

I intend to discuss such topics, as well as to report some recent work on combining paraconsistency and fuzziness.

By seeing vagueness as 'overdetermination of truth', instead of 'underdetermination of truth', M.E. Coniglio, F. Esteva and L. Godo define and axiomatize in [5] a family of extensions of MTL that preserve degrees of truth (called "degree-preserving"). Contrary to the truth-preserving fuzzy logics, the degree-preserving fuzzy logics turn out to be a class of LFIs that do not satisfy the law of excluded middle (in the sense that $\varphi \vee \neg\varphi$ is not necessarily a valid schema). Such extensions of MTL are thus paraconsistent fuzzy logics that enjoy most properties of general LFIs.

This line of thought gives formal substance to the concept of "paraconsistent vagueness" whose provenance can be found in previous papers such as [4], [6] and [10], and connects fuzziness and paraconsistency in a novel and promising way.

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5.6 The Paraconsistent Logic of Quantum Superpositions

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Physical superpositions exist both in classical and in quantum physics. However, what is exactly meant by 'superposition' in each case is extremely different. In this paper we discuss some of the multiple interpretations which exist in the literature regarding superpositions in quantum mechanics. We argue that all these interpretations have something in common: they all attempt to avoid 'contradiction'. We argue in this paper, in favor of the importance of developing a new interpretation of superpositions which takes into account contradiction, as a key element of the formal structure of the theory, "right from the start". In order to show the feasibility of our interpretational project we present an outline of a paraconsistent approach to quantum superpositions which attempts to account for the contradictory properties present in general within quantum superpositions. This approach must not be understood as a closed formal and conceptual scheme but rather as a first step towards a different type of understanding regarding quantum superpositions.

5.7 From Possibility theory to Paraconsistency

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Possibility theory is the simplest of uncertainty theories devoted to the modeling of incomplete information. It handles possibility and necessity measures instead of probability. As such it has connections to Kleene logic and epistemic logic as much as to probability theory. Recently a simple information logic called MEL, constructed as a two-tiered propositional logic, has been devised to provide a logical version of Boolean possibility theory. It can be viewed as a fragment of the KD logic, but with a much simpler semantics in terms of subsets of interpretations representing the epistemic states of an agent. Kleene and Łukasiewicz logics can be embedded in MEL, understanding

the third truth-value as *unknown*, letting modalities appear in front of literals only, and viewing three-valued valuations as partial Boolean models, in which MEL formulas can be evaluated.

When the third truth value refers to both true and false at the same time, the embedding of some three-valued paraconsistent logics seems to work, just altering the designated truth-value. Alternatively, we can consider a society semantics setting, where information revealed by the agents is possibly conflicting, each agent's information being encoded in MEL. Then, interpreting the formula $\Box\alpha$ as the assertion of proposition α by at least one agent, an extension of MEL is obtained which has axioms of the EMN non-regular modal logic, and which can encode Belnap's four epistemic truth-values. It can be shown that a possible semantics for this logic is in terms of general uncertainty measures, albeit expressible as the eventwise maximum of necessity measures, one per agent. When each agent is totally informed, we get a modal logic that is unusual in the sense that necessity modalities distribute over disjunctions instead of conjunctions. An equivalent translation into MEL can be obtained by exchanging the role of possibility and necessity modalities, highlighting the perfect symmetry between three-valued logics of contradiction and three-valued logics of incomplete information.

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5.8 Consequence-Inconsistency Interrelation: Paraconsistent Logics

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Classically the notions of consequence and inconsistency are interwoven. That is, considering one as the primitive notion the other can be derived. This equivalence depends on the notions of absolute inconsistency and negation inconsistency, which are equivalent in classical scenario. Absolute inconsistency states that given any formula α and its negation $\neg\alpha$, $\{\alpha, \neg\alpha\}$ yields any formula β. On the other hand, according to the notion of negation inconsistency, a set is said to be inconsistent if for some formula α, both α, $\neg\alpha$ follows from the set. In the context of paraconsistent logics this equivalence between absolute inconsistency and negation inconsistency does not work. So, the interrelation between consequence and inconsistency in the context of paraconsistent logics seems to be an interesting direction to be explored. In this presentation we shall concentrate on different fragments of a propositional language, and explore the connection between the notion of consequence and inconsistency where the consequence is non-explosive, i.e. it is not that for any α, $\{\alpha, \neg\alpha\}$ yields any β. We shall see that a relativized notion of inconsistency may help to retain the notions consequence and inconsistency interwoven in the context of paraconsistent logics too.

5.9 Restricted Quantification in Paraconsistent and Other Nonclassical Logics

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In many non-classical logics, both paraconsistent and otherwise, restricted quantification does not work very smoothly: while the logic may have a conditional '$\rightarrow$' that is well-behaved in many ways, the obvious definition of universal restricted quantification in terms of it (viz. $(\forall x)(Ax \rightarrow Bx)$) will typically lead to the failure of many highly desirable laws. This creates some pressure toward accepting classical logic.

On the other hand, there are some strong pressures toward non-classicality. To my mind the most compelling are to satisfactorily handle vagueness, and (even more) to satisfactorily handle the semantic paradoxes. At least when restricted quantification is ignored, it seems possible to satisfactorily handle both, either in a paraconsistent logic or a paracomplete one; there seems to be no decisive argument for choosing between paraconsistent and paracomplete treatments. But this needs to be reconsidered when considerations involving restricted quantification are taken into account: indeed, no currently published logic for vagueness or the paradoxes of either the paraconsistent or paracomplete sort handles restricted quantification at all well.

This paper will present a new approach to handling restricted quantification in paraconsistent and paracomplete logics. I will first show that the most obvious laws of restricted quantification can be validated in a relatively neutral framework that is both paracomplete and paraconsistent (i.e., no excluded middle and no explosion). I will then consider two strengthenings: (i) adding excluded middle but retaining paraconsistency; (ii) adding explosion but retaining paracompleteness. But it turns out that only the second is compatible with naive truth (or even, the Tarski biconditionals). One can modify the first to make it compatible with naive truth, but only by weakening the laws of restricted quantification.

It will be seen that there is a principled reason why the strong laws for restricted quantification that the account shows to be possible on option (ii) are unattainable in *any* paraconsistent theory that keeps excluded middle, or indeed on any version of paraconsistency that allows for the acceptance of dialetheia. While this doesn't undermine some of the motivations for paraconsistent logic, it does tend to diminish motivations based on treating the paradoxical sentences or "borderline case" sentences as dialetheia.

5.10 The many classical faces of quantum logic

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Logic is crucial to the verification and design of algorithms and protocols. But in the case of quantum computer science, the appropriate logic remains mysterious. Much like paraconsistent logic, quantum logic has to deal with overcoming counterintuitive phenomena. One such symptom is that standard (categorical) logic degenerates to modal logic in the quantum case. Luckily, logic also suggests a way out: much about a quantum system is captured by the collection of its classical subsystems. Hence quantum logic may be regarded as consisting of many classical faces. This leads to insights in the very foundations of quantum mechanics. I will survey this exciting development.

5.11 Modal logics connected to Jaśkowski's logic D2

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In the presentation we summarise results concerning modal logics defining the discussive logic $\mathbf{D_2}$ as well as modal logics defining the discussive consequence relation.

Jaśkowski's discussive logic $\mathbf{D_2}$ was formulated with the help of the modal logic **S5** as follows (see [1, 2]): $A \in \mathbf{D_2}$ iff $\ulcorner \Diamond A^{\bullet} \urcorner \in \mathbf{S5}$, where $(-)^{\bullet}$ is a translation of discussive formulae (the set of all discussive formulae is denoted by $\mathrm{For}^{\mathrm{d}}$) into the modal language. We say that a modal logic $\boldsymbol{L}$ *defines* $\mathbf{D_2}$ if, and only if, $\mathbf{D_2} = \{A \in \mathrm{For}^{\mathrm{d}} : \ulcorner \Diamond A^{\bullet} \urcorner \in \boldsymbol{L}\}$. In [8, 3] were respectively presented the weakest normal and the weakest regular logic which: (†) have the same theses beginning with '$\Diamond$' as **S5**. Of course, all logics fulfilling the condition (†), define $\mathbf{D_2}$. There is also a general method (see [5]) which, for any class of modal logics determined by a set of joint axioms and rules, generates in the given class the weakest logic having the property (†). Thus, for the class of all modal logics we obtain the weakest modal logic which owns this property. On the other hand, applying the method to various classes of modal logics, e.g. rte-logics, congruential, monotonic we obtain the weakest logic in a given class, defining $\mathbf{D_2}$.

Although Jaśkowski's logic $\mathbf{D_2}$ is usually understood as a set of discussive formulae, one can also define a consequence relation ($\mathbf{D_2}$-consequence) which refers to the paper [1]. After all, the logic $\mathbf{D_2}$ was meant to express a consequence relation. Similarly as the very logic $\mathbf{D_2}$, the $\mathbf{D_2}$-consequence is also defined with the help of the modal logic **S5**. A question of founding other than **S5** modal logics which define the consequence relation, arises. In [4] there are given such logics. Moreover in [6] a general method of founding modal logics which also allow to define the $\mathbf{D_2}$-consequence, is given.

Finally, there are also known (see [7]) the weakest ever modal logics defining: the logic $\mathbf{D_2}$ and the $\mathbf{D_2}$-consequence. For these two logics the condition (†) is not fulfilled.

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5.12 None of the Above: the Catuṣkoṭi in Indian Buddhist Logic

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The catuṣkoṭi (*Greek*: tetralemma; *English*: four points) is a venerable principle of Indian logic, which has been central to important aspects of reasoning in the Buddhist tradition. What, exactly, it is, and how it is applied, are, however, moot though one thing that does seem clear is that it has been applied in different ways at different times and by different people. Of course, Indian logicians did not incorporate the various interpretations of the principle in anything like a theory of validity in the modern Western sense; but the tools of modern non-classical logic show exactly how to do this. In this talk, I will show how.

I will approach the matter chronologically, interlacing philosophical and technical material, as appropriate. The point of the exercise to show how the history of (Buddhist) philosophy and the techniques of contemporary non-classical logic can profitably inform each other. Positions that one might take to be unintelligible can be shown to be perfectly coherent with the aid of these techniques; conversely, the positions may themselves suggest the development of new logical techniques.

6 – Talks of Contributing Speakers

6.1 Is it possible to perceive contradictions?

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Metaphysical dialetheism hold that there are true contradiction, or at least that it is possible for there to be true contradictions. According to a metaphysical dialetheist version of the correspondence theory of true due to JC Beall, there is nothing to prevent a positive fact and its corresponding negative fact from both obtaining in the same world. But, when we perceive, we can see that something is the case. Can we also perceive that something is not the case? Many have thought not. For example we cannot see that something is not green. Any judgment to the effect that it is not green has to be added to what we see by inference. This, according to Graham Priest is false. With a special pair of glasses that have a red filter on one lens and a green filter on the other we have the experience of seeing everything as red and as green. Priest's argument is the following: it might be said that being red and green is not a contradiction. But it is: red and green are complementary colours. It is, hence, a conceptual impossibility for something to be both colours. So, we should see directly that something is not green because, something that is red and green, is green and not green. But, I suggest, if we follow this, that is, if we can see something that is red and green, we also can see red and not red. But is it possible to see the same thing as red and not red, and simultaneously, as green and not green? Things get complicated, praeter necessitatem.

To perceive a contradictory scenario is to represent simultaneously a positive datum and a negative one that negate the first one. That is, the same thing should be represented as present and absent. If we put it in these words it's more difficult to find coherent examples. To say, as dialetheists say, that, at the instant a man leaves the room , he is neither inside it nor outside it, and to represent this as a case of observable contradiction, or, to avoid to postulate a gap between truth value that would introduce a third one, to say that he is both in and out, it only means to confuse an instantaneous state with a state of affairs. They are looking in the wrong place and missing the obvious. Some requirements are necessary: we have good reasons to exclude visual illusions, instantaneous states, impossible pictures (e.g. Escher's lithographs), inconsistent descriptions, in short everything is theory laden, that is, phenomena without a contradictory content within it. If we adopt Dretske's classical terminology, we can realize how deep are confusions about our problem. If we focus attention on the distinction between epistemic seeing ('seeing that') and non-epistemic seeing, we observe the ambiguity behind the debate over whether conceiving p entails the possibility of p.

Epistemically conceiving p does not entail the possibility that p. But non-epistemically conceiving p does entail that p is possible. Conceivability is not a mere a priori matter.

We perceive phenomenal realities, and my claim has been that to perceive a contradiction is to perceive the same thing as present and absent. Then let us for argument's sake suppose that image-experience is like perceptual-experience in being experienced as an observational-experience with a phenomenal object similar to the real one and experienced as a distal object in the flow of information. Imagining intends absent objects; perceiving intends present objects â€" same objects, different intentional relation. So, is it possible to see something and, in the same time, to image something else that negate it? Here with 'to image' I simply mean an episode of imagery, a mental image. Or, starting from a more basic question, is it possible to see and to image the same thing? For example, while we are looking at our mother we can try to visualize her face, and we need the same content in the very same way. But this exercise is not easy at all. It is known that there is overlap in the regions of the brain that are activated in seeing and visualizing. According to Kosslyn the same cerebral mechanism in our neuroanatomy must be involved, the Visual Buffer. In Zettel, Wittgenstein says: while I am looking at an object I cannot imagine it (§621). This means that I cannot imagine the very object I am looking at. I can surely be looking at my mother from the back, not even realize I am looking at my mother, and still imagine her from the front. De re seeing does not prevent de dicto imagining. The point is that it is not possible for our cognitive resources to represent the same thing as absent and present. To come closer to our problem, we can consider a metaphysical way of stating the law of non-contradiction. It would be a second order one:

$$\forall x \, \forall P \, \neg(P(x) \wedge \neg P(x))$$

But, in order to demonstrate that our second question, the basic one, is more difficult than the first one, we need to quote Aristotle, when he says, in the Metaphysics: the same attribute cannot at the same time belong and not belong to the same subject in the same respect (1005b19–22). So we have a genuine contradiction when the same property is predicated and not predicated of an object. Therefore, Aristotle explains that we do not get the point insofar as we adopt a distinction of respects, a different parameter, that is, a plea of ambiguity. If we put together the psychological argument, about how the mind works, with the metaphysical one, about how the world can't be, things get complicated for dialetheists. The impossibility of an observable contradiction, that is, an incoherence in its ontological status, does not seem to be questionable by the dialetheists anymore. However, we haven't negated the conceivability of contradictions. To appreciate this point, it is sufficient to recall the classical formulation for possibility operator:

$$\Diamond A \stackrel{\text{def}}{=} \neg\Box\neg A$$

The problem is how to intend the modal operators? I suggest that our knowledge of conceiving is a priori knowledge, and that knowledge of metaphysical possibility is at least partly a posteriori in nature.

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6.2 Many-valuedness, paracompleteness and paraconsistency in a 3-oppositional quadri-simplex of sheaves

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A standard way of defining paraconsistency consists in seeing it as the possibility of having, inside a formal system, "*contradictions* without triviality". Slater's harsh attack against the very idea of paraconsistency, consequently, relies on a merciless analysis of the notions of contradiction, contrariety and subcontrariety, which pertain to the old "square of opposition". This has generated several reactions from paraconsistent logicians. Among these, the 2-fold one of Beziau consisted, in one of its halves, in studying the difference between the notions of logical square and hexagon and in claiming that a related "solid of opposition" justifies paraconsistency. Beziau's paper has *de facto* generated a renaissance of the studies on the geometry of oppositions, which has led Moretti to highlight a mathematical structure common to all n-opposition hyper-solids (including squares and hexagons): that of "oppositional (or logical) bi-simplex (of dimension m)". As a complement, Pellissier has developed a mathematical technique, based on set-theoretical tools, for allowing manipulating usefully any bi-simplex. Later, in order to extend the geometry of oppositions, which so far was only 2-valued (however big the n-opposition and its closure), to arbitrary *many*-valued universes, Moretti has proposed a geometrical method for generalising the notion of (oppositional) bi-simplex (of dim. m) into that of arbitrary (oppositional) *poly*-simplex (of dim. m). But then the aforementioned set-theoretical method suffices no more for decorating such poly-simplexes. So, in recent times, Pellissier has developed a suited new mathematical technique, based on sheaf-theory, for allowing making an explicit use of the notion of oppositional poly-simplex (of dim. m). Keeping paraconsistency in focus, in this paper we use this sheaf-theoretical technique for studying the "oppositional closure" of the quadri-simplexes of dimension 2, that is, the quadri-simplicial counterpart of the standard (i.e. bi-simplicial) logical hexagon. Among several other results, we show that this leads to six logical hexagons. In particular, as for their logic, one of them is the usual classical one, whereas two are paracomplete, two are paraconsistent and the last one is both paracomplete and paraconsistent. It is in the light of these new results that we come back to the issue about Slater's attack to paraconsistency.

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6.3 Game Semantics and Paraconsistency

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In this paper, I offer an extension of game theoretical semantics for negation for both classical and non-classical logics. I motivate it by discussing the meta-game theoretical role of negation in game theoretical semantics.

Game semantics suggests role switching for negation, and insists on using the negation normal form for the formulas to maintain the intuition. However, when it comes to non-classical logics, role switching idea does not immediately carry over, and the need for a broader idea becomes a necessity. In this paper, we start by offering additional intuitive rules for the negated formulas for the classical logics and show the correctness of the game semantics.

The main contribution of this paper is to suggest a game semantics for negation in paraconsistent logics. A simple way to introduce non-classicity to game semantics is to expand some conditions of the game as follows. In a game with Abelard and Eloise, we can have the following situations:

- ✦ Abelard and Eloise may both win.
- ✦ Abelard and Eloise may both lose.
- ✦ Eloise may win, Abelard may not lose.
- ✦ Abelard may win, Eloise may not lose.
- ✦ There may be a tie.

We introduce a formalism that enables us to discuss the extended winning conditions for games. The formalism we adopt here is Priest's Logic of Paradox. The logic of paradox introduces an additional truth value called *paradoxical*, that stands for both true and false.

The introduction of the paradoxical truth value requires an additional player in the game, and we call him *Astrolabe* (after Abelard and Heloise's son). The interesting observation here is that a win for Astrolabe does not imply a win for Abelard or Eloise. We will suggest Astrolabe as the *parallel player*, and suggest a game semantics for this case.

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6.4 With Librationism from Paraconsistency to Contrasistency, Incoherency and Complementarity

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More information on the foundational system Librationism, now denoted £, is available in [1]. More recent developments such as work describing its interpretation of ZF if ZF + "there are omega inaccessible cardinals" is consistent are available from the author. In unpublished superseded accounts £ was understood as a non-adjunctive paraconsistent system. It turns out that we in our reasoning from the outside about £ best think of connectives as acting upon valencies which are the ordinals where a sentence holds in the Herzbergerian style semi inductive semantics. The valor of a sentence is the least upper bound of its valency. The contravalence of a sentence is the closure ordinal minus the valency of that sentence, and the ambovalence of two sentences is the intersection of their valencies. Similar definitions introduce the concepts of velvalence, subvalence of ... under ..., and homovalence for disjunction, subjunction (material conditional) and equijunction (material biconditional). A sentence is true iff its valor is the closure ordinal, and a sentence is false iff its negjunction (negation) is true. Connectives of £ are valencyfunctional, and accordingly also truth functional in the special case for non-paradoxical sentences. A sentence dictates its valor, and its valency is the way the valor is dictated. We take two sentences to contradict each other iff they are contravalent and dictate different values. Two sentences are complementary iff they are contravalent and dictate the same, i.e. thence the closure ordinal. Let r be Russell's set $\{x \mid x \notin x\}$ and *Russell's sentence* be $r \in r$. Russell's sentence and its negjunction dictate the same in opposite ways. Let us agree that a theory is contrasistent iff it has a thesis A as well as its complementary negjunction $\neg A$ as a thesis. We take a theory to be inconsistent iff it has a thesis of the form A and not A. Inconsistent theories of any interest will also be contrasistent. £ is contrasistent, but not inconsistent. Moreover, unlike in typical paraconsistent approaches, ex falso quodlibet as well as all other theses of classical logic remain theses of £, and £ has no thesis which contradicts classical logic. £, which is a super (semi) formal system is not recursively axiomatizable, but a lot of informative prescription schemas ("axiom schemas") and regulations ("inference rules") are isolated. [1] established that £ is stronger than the Big Five of the Reverse Mathematics Program.

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6.5 Mission Impossible: A Dialetheic Solution to Curry's Paradox

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Curry's paradox has the dubious honor of being *insoluble* by dialetheic methods. A dialetheic solution posits that a given paradoxical sentence is both true and false, but it is allegedly impossible for a Curry sentence to be true. Many consider this the single, greatest problem for dialetheism, as it dashes any hope of a unified dialetheic solution to the paradoxes.* In this paper, I argue that the impossible is, in fact, possible: Curry sentences can be true, and there can be a unified dialetheic solution to paradox.

My approach revolves around an idea originally due to Bradwardine: that paradoxical sentences express multiple, conflicting propositions.† For instance, the Liar expresses both the proposition *that the Liar is false* and the proposition *that the Liar is true*. The content of paradoxical sentences is overdetermined in a way that contrasts with ordinary, non-paradoxical sentences. A typical Curry sentence has the following form:

If this sentence is true, it follows that everything is true.

I posit that this sentence expresses the proposition *that the Curry sentence entails everything* and the proposition *that the Curry sentence is true*. It is clear why such a sentence apparently cannot be true: it seems that if it were true, then everything would be true, which is absurd.

On my analysis, the error is that this reasoning conflates two distinct kinds of conjunction. In one sense, viz. the *extensional* notion of conjunction, we are indeed committed to 'all' of the conjoined contents of the things we assert, but in this sense *modus ponens* is invalid. In another sense, viz. the *intensional* notion of conjunction, it is true that *modus ponens* holds, but in this sense we are not necessarily committed to arbitrary conjunctions of things we assert. What makes this analysis of the paradox possible is that we assume a structurally *non-contractive* framework.‡ Briefly, if $\mathbb{S}$ is our theory of truth and κ as a Curry sentence, there are two reading of the following inference in a non-contractive framework.

$$\dfrac{\dfrac{\mathbb{S} \vdash \mathrm{T}(\ulcorner \kappa \urcorner)}{\mathbb{S} \vdash \mathrm{T}(\ulcorner \kappa \urcorner) \to \bot}\,(\text{T-out}) \qquad \mathbb{S} \vdash \mathrm{T}(\ulcorner \kappa \urcorner)}{\mathbb{S}, \mathbb{S} \vdash \bot}\,(\text{MP})$$

The last step is invalid on the extensional reading of premise combination; it is valid on the intensional reading, but nothing follows directly from this about the triviality of the theory $\mathbb{S}$.

*For elaboration on this line of criticism see, e.g., Goodship (1996) and Whittle (2004).
†An illuminating account of Bradwardine can be found in Read (2002).
‡Such frameworks have been explored by, e.g., in Paoli (2007).

When we say that it is impossible for a Curry sentence to be true, we conflate extensional and intensional notions of conjunction. If sentential content were closed under intensional conjunction, then we could make no coherent sense of the idea that a Curry sentence is true, but on the assumption that paradoxical sentences express conflicting propositions, we should expect their contents *not* to be closed under intensional conjunction. Thus, in a non-contractive framework a Curry sentence can be true, and *a fortiori* we can apply a dialetheic solution to it.

Once this basic point is made clear, it raises many questions in need of further elaboration. For example, is there an informal distinction, perhaps something doxastically or semantically normative, which tracks the distinction between extensional and intensional conjunction? Even if it is possible to make 'logical space' for a dialetheic treatment of Curry's paradox, would such a posit *solve* the paradox in the same way that dialetheism solves the Liar directly? I will address several of these questions in my talk.

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6.6 The Basic Logic of Consistency: syntax, semantics and philosophical motivations

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Paraconsistent logics are non explosive logics. So, as is well known, a theory can be contradictory without being trivial if its underlying logic is paraconsistent. A central question for paraconsistent logics is: what does it mean to accept a contradiction? Or more precisely: saying that a theory deduces a pair of sentences A and not A is the same of saying that A and not A are both true? Definitely not! The acceptance of a contradiction may be taken as a provisional state, a kind of excessive or defective information that should, at least in principle, be eliminated by means of further investigation. According to this view, contradictions have an epistemological rather than an ontological character. Logics of Formal Inconsistency (LFIs) are a family of paraconsistent logics that have resources to express the notion of consistency within the object language, and thus recover the full power of classical logic for consistent sentences. LFIs are able not only to distinguish triviality from contradiction, but also non contradiction from consistency. In this talk we present a LFI that we call the Basic Logic of Consistency (BLC).

BLC changes the requirements put by Newton da Costa with respect to paraconsistent systems: instead of having everything that could be added without validating explosion and non contradiction, BLC has the minimum necessary to restore classical logic for consistent formulas. Its deductive system is obtained by adding to introduction and elimination rules for $\to$, $\wedge$ and $\vee$ an explosion rule restricted to consistent sentences (EXP) $\circ A$, A, $\neg A$ / B and excluded middle as an axiom (EM) $A \vee \neg A$. A sound and complete valuation semantics for BLC is presented.

We will show that BLC fits the idea that contradictions have epistemological character. The values 1 and 0, attributed to a formula A, may be interpreted, respectively, as 'there is some evidence that A is the case' and 'there is some evidence that A is not the case'. Thus, a contradiction $A \wedge \neg A$ means only that there is some evidence that A is the case and not A is the case, a situation very common in empirical sciences. Consistency is primitive and not defined in terms of negation. Its meaning is thus established from outside the formal system, which opens up the possibility of interpreting it in different ways. $\circ A$ may be understood as 'the truth value of A has been conclusively established'. So, we may have $\neg(A \wedge \neg A)$ without $\circ A$, for example, a circumstance such that there is evidence only for A, but the truth value of A has not been decided yet. However, a central problem for an intuitive interpretation of LFIs remains: the failure of the replacement theorem with respect to disjunction and conjunction in the scope of negation. We also discuss some alternatives to circumvent this problem.

6.7 Was Heidegger a Dialetheist?

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An innovative element in Graham Priest's philosophy is the systematic investigation of the limits of our concepts. In *Beyond the Limits of Thought*, Priest identifies those boundaries in the limit of what can be expressed, conceived and known. The main idea is that such limits are dialetheic and therefore they emerge as the eminent locus of true contradictions.

One of the examples proposed by Priest in order to present a case study concerning the limits of expressibility is the problem of "nothingness" discussed by Martin Heidegger. Significantly Heidegger's discussion of the issue of nothingness take place just before his *Kehre*, the turning point in Heidegger's thought. By reading nothingness not as a quantifier-phrase but as a noun-phrase (then, the nothingness with the article in front of it), it is not possible to say anything about nothingness itself. Since nothingness is defined as the pure absence of all objects and properties, to say something about nothingness would mean to turn nothingness into an object with properties. Nothingness is not expressible. It seems that the only possible solution is to consider nothingness as an impossible object with contradictory properties.

The present work will propose an interpretation of Heideggerian philosophy after the *Kehre* claiming that the late Heidegger is a dialetheist. Here the *Kehre* is interpreted as the point after which Heidegger not only overcomes the principle of non-contradiction but also accepts true contradictions. It will be shown that the same problem faced in defining nothingness can be found in defining being. For Heidegger, *being* is expressed by the general form of the statement X is [y] (where the y can be omitted) and it is both the "being of predication" and the "being of existence". It will be shown that the question of being (What does *being* mean?) leads to the same aporetic situation that we face with nothingness. Indeed, *being* cannot be simply considered a general predicate. Following both the neo-platonic tradition (with Plotinus) and Meister Eckharts philosophy, Heidegger introduces the so-called *ontological difference* theorizing that, if other properties (being red, being at, etc.) can be entities as well because they have at least the property of being a property, being itself is not an entity. As nothingness, being is not expressible. Two main arguments will be introduced. The first one will use the syllogism proposed by Priest in order to justify the identity between nothingness and being: it will be argued that, given that nothingness and being are the same, then being cannot be an entity because nothingness is not an entity either. Secondly, the ontological difference will be restated in the light of the strategy that Gottlob Frege used to save the unity of propositions. As Frege distinguished between objects (that are saturated) and concepts (that are unsaturated), Heidegger distinguished between being itself and all the other properties or entities. Finally, it will be shown how Heidegger tries not to reject contradictions and to deal with contradictory objects (as being and nothingness) by introducing a new definition of negation, in the thirty-sixth paragraph of the Contribution to philosophy. It will be stated that the Heideggerian negation not only has important implications on the definition of truth (that is not merely the negation of falsity anymore) but it could also provide a better understanding of the contentious topic of paraconsistent negation.

6.8 Consistency operator and 'just-true' operator in paraconsistent Weak Kleene logic

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It is well known that if the Logic of Paradox (LP), developed by Graham Priest is expanded by the consistency operator $\circ$, then classical negation and a detachable material conditional are definable in it, and the resulting logic is known as LFI1 designed and developed by Walter Carnielli, João Marcos and Sandra de Amo. Furthermore, we may easily verify that 'just-true' operator $\circledast$, understood as follows, has the same logical power in the context of LP:

ϕ	$\circledast\phi$
t	**t**
b	**f**
f	**f**

Indeed, two expansions of LP enriched by $\circ$ and $\circledast$ becomes equivalent, since $\circledast\phi$ can be defined as $\circ\phi \wedge \phi$, and $\circ\phi$ can be defined as $\circledast\phi \vee \circledast{\sim}\phi$ where $\sim$ is the paraconsistent negation. Therefore, if we consider transparent truth theory based on these two expansions of LP, then we end up in triviality.

On the other hand, it is widely known that LP can be regarded as paraconsistent strong Kleene logic since we obtain LP by changing the designated values of strong Kleene logic. But there is also weak Kleene logic known in the literature, and so we may consider its paraconsistent version as well. We refer to this logic as WKL_2.

Based on these, the aim of the paper is to examine the two operators $\circ$ and $\circledast$ in the context of WKL_2. The two main results are the following: (i) the expansion of WKL_2 by the consistency operator is strictly weaker than the expansion of WKL_2 by 'just-true' operator; (ii) classical negation and a detachable material conditional are definable in the expansion of WKL_2 by 'just-true' operator. The paper will also present some proof theories for the two expansions of WKL_2, and prove the completeness results. In particular, the expansion of WKL_2 by 'just-true' operator can be placed in the context of Logics of Formal Inconsistency (LFIs) developed by Carnielli, Marcos, De Amo and Coniglio. We will refer to this expansion as LFI3, and compare LFI3 with LFI1 and LFI2 which share the truth tables for paraconsistent negation and the consistency operator, but differ in the truth tables for conjunction, disjunction and classical conditional.

The main results also open interesting insights in the debate on dialetheism and semantic closure, which we briefly discuss in the final part of the paper. The key point is that, since trivialization of a semantically closed paraconsistent logic usually depends on the presence of classical negation, we may expect that no trivializing Liar can be constructed in WKL_2 plus $\circ$ together with a transparent truth-predicate (a combination that proves problematic for LP, as we mentioned above). WKL_2 would thus prove more resistant to trivialization than LP, with respect to the addition of the consistency operator. At the same time, definability of classical negation in WKL_2 plus $\circledast$, i.e. LFI3, makes that a trivializing Liar can be constructed for the logic; this in turn limits the extent of WKL_2's resistance to trivialization.

6.9 Identity Through Time and Contradiction

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A certain amount of contemporary work in non-classical logics would seem to based contradiction in reality. Indeed, as Graham Priest wrote, "if some contradictions are true, then the world must be such as to make this the case" (Priest, 2006, 299). I would like to consider the controversial idea that contradiction arises in reality in the identity through time — or 'persistence through time'.

At first glance, the contradiction within persistence can be described as the instantiation of inconsistent properties. For instance, a book persists or survives through change and so possessed the incompatible properties of being open and being shut — since change require incompatible properties. Hence the book is open and shut (assuming an atemporal instantiation) (Haslanger and Kurtz, 2006). The problem of identity through time is to connect the intuition of identity of a thing and the incompatible change of this same thing. Since persistence does not satisfy the principle $A_{t_1} \neq \neg A_{t_2}$, the main contemporary ontologies tried to avoid this contradiction by different means. However, each of them have serious worries which encourage me to shift my inquiry into another way.

So in this talk, my aim is to investigate another approach by trying out the hypothesis that contradiction within persistence must not be avoided, but rather taken seriously. To this end, I will develop (1) the 'problem' of persistence as it is usually understood and (2) the idea of inconsistent motion. I will suggest a link between the notions of the Leibniz Continuity Condition and the Spread Hypothesis (Priest, 2006) and the notion of identity used within the metaphysics of persistence, i.e. diachronic identity. I will show that (1) there exists a link between the contradiction in motion (Priest, 2006, and Mortensen 2011) and the contradiction about properties within persistence, and that (2) the contradiction in motion — which is a particular case of change — is the condition and the ground of the contradiction in diachronic identity — which need change.

The result, as disturbing as it can be, will be a new way to conceived identity through time. Since identity through time could be seen as an identity relation which assumed change in properties, the identity relation $A_{t_1} = A_{t_2}$ is nothing more than the diachronic identity relation which we called the 'C-relation of identity'. In other words, if non-contradiction ground identity at a time — a numerical identity $A_{t_1} = \neg A_{t_2}$ —, contradiction ground and define identity through time — as an 'identity-and-difference relation'. It can be shown that contradiction is a necessary and sufficient condition for identity through time (since contradiction is necessary to defined change and arises only from continuous state which is a principle of unity of things).

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6.10 On a paraconsistentization functor

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We show how to define a paraconsistentization functor able to convert any explosive logic into a paraconsistent logic. Our main tools to realize this task are category theory and abstract logic. Previous works in this direction include [1] and [2].

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6.11 A unified proof-theoretic approach of partial and paraconsistent three-valued logics

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On the sidelines of classical logic, many three-valued logics have been developed. Most of them differ in the notion of logical consequence, which is sometimes partial, sometimes paraconsistent, or in the definition of the logical connectives. Indeed, from a semantic point of view, a plurality of three-valued logics can be distinguished by changing the designated values or the truth functions for some of the logical connectives.

This paper aims, firstly, *to provide a unified proof-theoretic approach of the three-valued logics* and, secondly, *to apply this general theoretical framework to several well-known partial and paraconsistent three-valued logics*. The proof-theoretic approach to which we give preference is sequent calculus. Insofar as it consists in developing a unified sequent calculus for the investigation of these logics, our talk faces two issues: the interpretation of the notion of logical consequence and the interpretation of the logical connectives.

To address the diversity of the interpretations of logical connectives, we distinguish between two languages, the surface language and the deep language. We then show that any logical connective of the surface language can be translated into the deep language, which is functionally complete with regard to the given semantics. On the other hand, to address the distinction between the partial and the paraconsistent notions of logical consequence, we propose a single hypersequent-inspired calculus for the deep language. In this way, *a sequent calculus and a uniform proof-search method are provided for any three-valued logic.*

Starting with this general framework, we study the relationships between several well-known three-valued logics such as Kleene's strong three-valued logic (K_3), Łukasiewicz's three-valued logic ($Ł_3$), Heyting's three-valued logic (G_3), the maximal weakly-intuitionistic three-valued logic (I^1), Priest's logic of paradox (LP), Dunn's R-mingle three-valued logic (RM_3), Brouwer's three-valued logic (G_3^*), the maximal weakly-Brouwerian three-valued Logic (P^1), etc. By means of a proof-theoretic characterization of these logics, *we investigate the notions of expressive power, duality and cut-redundancy specific to these logics.* We also provide a general Rasiowa-Sikorski-style proof for the soundness and completeness of the sequent calculi associated with these three-valued logics.

6.12 Paraconsistent degree-preserving fuzzy logic

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Non-classical logics aim to formalize reasoning in a wide variety of different contexts in which the classical approach might be inadequate or not sufficiently flexible. This is typically the case when the information we want to reason about is incomplete, imprecise or contradictory.

On one hand, **fuzzy logics** have been proposed as a tool for reasoning with imprecise information. Their main feature is that they allow to interpret truth in a linearly ordered scale of truth values which makes them specially suited for representing the gradual aspects of vagueness. Originating from fuzzy set theory, they have given rise to the deeply developed area of mathematical fuzzy logic (MFL) [3]. Particular deductive systems in MFL have been usually studied under the paradigm of *truth-preservation* which, generalizing the classical notion of consequence, postulate that a formula follows from a set of premises if every algebraic evaluation that interprets the premises as true also interprets the conclusion as true. An alternative approach that has recently received some attention is based on the *degree-preservation* paradigm [1], in which a conclusion follows from a set of premises if for all evaluations its truth degree is not lower than that of the premises. It has been argued that this approach is more coherent with the commitment of many-valued logics to truth-degree semantics because all values play an equally important rôle in the corresponding notion of consequence [5]. On the other hand, **paraconsistent logics** have been introduced as deductive systems able to cope with contradictions. Inconsistency is ubiquitous in many contexts in which, regardless of the information being contradictory, one is still expected to extract inferences "in a sensible way". One approach are the logics of formal inconsistency (LFIs) studied by the Brazilian school [2].

We have recently started an investigation of logical systems able to cope with vague and inconsistent information at once [4]. In this talk we will present our approach to this problem. We study paraconsistent fuzzy logics in the context of MFL. We argue that the appropriate paradigm for that is not the usual truth-preserving approach, but the degree-preserving one. We show that truth-preserving fuzzy logics are explosive, while under some conditions degree-preserving logics are not; we explore their paraconsistent features, give particular examples to illustrate them and characterize a family of LFIs inside fuzzy logics. Finally, since paraconsistency is always defined with respect to a particular negation connective, we explore alternative negations in fuzzy logics and their interplay with paraconsistency.

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6.13 Negation and the Metaphysical Foundations of Logic

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A comparison between Hegel's dialectics and dialetheism is fundamental in at least two senses. First, the dialetheic semantic of negation makes possible to put the double (dialectical) use of negation in the clearest terms. Second, as Apostel 1972 stresses, the clarification of the meaning of negation in the philosophy of logic cannot do without a consideration of the metaphysical foundations of logic. Hegel's reflections on negation are possibly the most in-depth analysis in order to explore the metaphysical implications of the "not" operator and similar devices of our languages. And this is a still controversial and discussed topic, for dialetheists. In my paper, I focus on the second point.

6.14 Strong Three-Valued Paraconsistent Logics

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We investigate three-valued paraconsistent logics in which both formulations of the principle of non contradiction are not valid, i.e. neither $a \wedge \neg a \vdash b$ nor $\vdash \neg(a \wedge \neg a)$ hold. Our study is based on structural consequence relations with classical conjunction and disjunction. We show that there are only two possible solutions and compare them. We discuss the question of molecularization and possible interpretations of the third value.

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6.15 Unscrambling the 'Copenhagen omelet' in paraconsistent terms

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"I want to know how Nature avoids contradiction."
— Niels Bohr

"Nature is earlier than man, but man is earlier than natural science."
— Von Weizsacker

I: Physical perspective of the problem

The outcome of the whole historical development of the language of standard quantum mechanics is described by E.T. Jaynes as follows: "But our present (quantum mechanical) formalism is not purely epistemological; it is a peculiar mixture describing in part realities of Nature, in part incomplete human information about Nature — all scrambled up by Heisenberg and Bohr into an omelette that nobody has seen how to unscramble. Yet we think that unscrambling is a prerequisite for any further advance in basic physical theory. For if we cannot separate the subjective part and an objective aspect of the formalism we cannot know what we are talking about, it is just that simple." [1]

This small passage calls for a clarification of issues centered around the relation between Language, Logic and Reality, including even (remotely) the way Human language — a very special feature of the Universe, evolved through history hosting often within it unqualified excess baggage. Mathematical language of quantum mechanics, irrespective of its social input characteristic of that period, is one of the latest creations by such ordinary language-users living in and grown up within a slowly varying part of our Universe.

This made quantum mechanics to develop within classical mathematical embryo — inevitably as a MIXTURE, endorsed by features/metaphysical presuppositions characteristic of ordinary language users. So the peculiarity of the "peculiar mixture", Jaynes referred to, is likely to be discussed in relation to this human-centric perspective as a whole. However, this is a huge topic and in this presentation we will try to get only to the grounds of some of the logic-philosophical aspects of this mixture that led to Jaynes's omelette and to understand what possible senses can be made of UNSCRAMBLING.

In fact we can decouple the process of development of the wave mechanical version of the language of quantum mechanics during 1924 to 26 in two stages:

1st) Capturing de Broglie relation (1924) formally within the framework of Schrödinger equation (a fragment of functional analysis) admitting implicitly a FAILURE of the law of excluded middle. And in that way effectively capturing a sense of failure of standard logic within a framework of mathematics faithful to standard logic!

2nd) Born's rule (1926) to talk EPISTEMICALLY about the Schrödinger equation leading to difficulties to talk about Individual quantum system with preexisting properties. In that way, Born rule can be understood to define the scope of semantics of a metalanguage.

So the 'peculiar mixture' Jaynes referred to is first of all about mixing a token of failure of standard logic with a fragment of functional analysis relying on standard logic itself, and then coupling this with Born rule, a formal recipe of the role of observer. We will discuss mainly the first part here — the way the wave-particle duality was made further formal sense in terms of failure of law of excluded middle expressed as superposition principle. While Einstein and early Schrödinger were famously skeptic about the language of quantum mechanics thus emerged, Bohr and his followers continued to present grounds of defense within the framework of Complementarity ever since 1927 contrasting it categorically from Contradictory. What is curious to note that, though Bohr's emphatic defense seems to be very much with a spirit of new mechanics, he was after all notoriously skeptic, more like a disguised Aristotelian, to make any ontological sense of the wave particle TOGETHERNESS or UNION of contradictory aspects!

"Even the mathematical scheme does not help." He lamented seeing contradiction to lurk behind, as reported by Heisenberg later (in an interview by Thomas Kuhn, 1963). "I want first to know how nature actually avoids contradiction."

So Bohr's recipe of 'scrambling' can be diagnosed to seek its prime justification partly to complementarity, or in other words relying on Nature's unquestionable ability (believed by Bohr and those who believed him!) to avoid contradiction.

Bohr spelled out his skepticism in more clear terms in these lines: "Complementarity denotes the logical relation of quite a new type, between concepts which are mutually exclusive, and which therefore cannot be considered at the same time — that would lead to a logical mistake — but which nevertheless must both be used in order to give a complete description of the situation." [2]

These lines, in spite of Bohr's appreciation of the need of a new logical framework, reveals his underlying commitments to classical Aristotelian logic, as the 'logical mistake' he seems to be scared about is a mistake from classical logical point of view not admitting contradiction. In fact, he never articulated clearly that, what formal sense can possibly be made of what he termed as new logical type — what constitutes the newness. So the suggestion to adopt the term complementarity as a token of avoidance of contradiction without any mathematical scheme (which of course he himself thought of no use!) served no more than a way to ensure only a semantic variation.

So one way to make sense of â€˜Unscrambling' seems to talk about in terms of possibility to read/interpret de Broglie's relation differently right from the beginning, unlike what happened historically and defended subsequently, probably admitting contradiction at the level of ontology more seriously than ever — de Broglie's relation as an instantiation of provision of nature's options (or preference?) to make choice on our part between contradiction and complementarity. Bohr would not possibly be happy with all these.

Put the question precisely — what a quantum mechanics would look like IF de Broglie relation were interpreted in terms of failure of the law of contradiction along with the law of excluded middle, but keeping the formal recipe of the role of observer (Bohr Rule) intact?

II: Can Paraconsistent Logic help?

This counterfactual question calls for the relevance of Paraconsistent paradigm as the paraconsistent logicians claim to take formally care of contradiction [3]. In fact the basic issue is to settle, first of all, given a handful of non-standard toolkits, where to make the non-standard surgery in the standard body of knowledge, though the fate is uncertain. Rather than reinterpreting, this is more about a question of considering possibility of reconstructing quantum mechanics in a way different from what actually happened.

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6.16 Paraconsistent Algebra of Sets, Information and the Bar-Hillel-Carnap Paradox

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The proposal of this paper is to provide a more detailed analysis of information theory focusing the Bar-Hillel-Carnap Paradox. Accordingly, we describe some aspects of semantic information theory and the relationship of this theory to problems involving different logical paradigms. This can be seen, for example, in the relationship between Łukasiewicz's three-valued logic and the so called Ulam's game. We describe a part of the theory of semantic information such as it is found in [1] and [3]. Then based on the logics of formal inconsistency presented in [2] and in the suggestion of treating inconsistent information from certain systems we generalized the notion of logical spectrum to non-classical cases. Our first approach to such spectra showed that it was possible to obtain a solution to the Bar-Hillel-Carnap Paradox. This is because the amount of information acquired in a contradictory context may not be the maximum, if the underlying logic is paraconsistent. Such a proposal, however, must be supported by appropriate formal examples. Therefore, the presentation of a structure as a paraconsistent algebra and an paraconsistent algebra of sets becomes relevant. In this respect, we present some candidates to fill these demands. We suggest a novel construction that allows us to view, propositional paraconsistent structures in terms of set theory. For this it was convenient to define a new combinatory operations in set theory that extends the set theoretic notion of power set. This contribution is important in that it paves the way for a new approach to formal paraconsistent systems as well as the basis for an unprecedented study of the theory of semantic information in a non-classical context.

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6.17 Is the disjunctive syllogism even a quasi-valid inference?

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One of the most common objections to paraconsistent logics is that they produce trivialism. According to this objection, if a single contradiction happens to be true, it immediately follows that everything is true (this is also called "explosion"), and that consequently logical discourse loses its main purpose, i.e. assessing the truth of determinate statements. The defenders of paraconsistency are well aware of this protest, and have no difficulties in answering it: they show that trivialism is not specifically the result of accepting true contradictions, but rather a consequence of combining these contradictions with logical tools that were conceived and used in consistent frameworks. Since it is more important, as far as paraconsistent logicians are concerned, to preserve the possibility of stating contradictions without explosion, than to keep old logical tools untouched, their strategy has so far consisted either in modifying these tools, or in giving them up.

Among these explosion-bringing tools, the rule of inference called "disjunctive syllogism" is the one I'm interested in. It can be written as follows:

$$\{\alpha, \neg\alpha \vee \beta\} \vdash \beta$$

As it can be seen, in case α turns out to be both true and false, both premises are then true, whatever β stands for; thus anything, any β can be logically deduced from a contradiction, as long as disjunctive syllogism remains valid. In chapter 8 of his book In contradiction, Graham Priest exposes his strategy regarding the disjunctive syllogism. Since this rule of inference is often used successfully in everyday-life, Priest recommends not to merely abandon it, but rather to circumscribe its legitimate use. Following what he calls "quasi-validity", that rule would still be valid in consistent contexts, but not when a contradiction arises. But it is not easy to tell the difference between these two situations : any clause according to which α is not contradictory might just as well be contradictory itself (which would lead again to explosion) ; in paraconsistency, nothing assures us definitely that it is not. And so he turns himself to a pragmatic approach, mobilizing the notions of rejection and acceptance, in order to avoid this difficulty.

In my talk, I would like to examine, and perhaps challenge, the view that the disjunctive syllogism may still be valid in a consistent "portion" of paraconsistent logics: instead of turning those logics into two-headed hydras, one of whose heads would allow contradictions, while the other wouldn't, abandoning the disjunctive syllogism might prove to be a more satisfactory solution for paraconsistency. I intend, then, to report and assess some important lines of argument that have been held against the use of that rule of inference, and might be able, after the inquiry, to legitimately propound such a solution to paraconsistent logicians.

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6.18 Consistency in Indian Logic

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In Indian intellectual tradition, Nyaya system as Logic and Epistemology occupies an important place that was held in high esteem. It is characterized as the lamp of all sciences, the resource of all actions and the shelter of all virtues. Gangesha Upadhyaya (1300 A.D.), the founder of Neo-Nyaya (Navya Nyaya Logic), acknowledges the importance of Old Nyaya of Gautama, who systematized the principles of correct thinking.

The aim of the present paper is to analyze and reconstruct, Nyaya theory of epistemic logic by way of considering form and content of correct inferences as distinguished from the patterns of incorrect inferences containing contradictions, triviality and other syntactical and semantical anomalies. Incidentally, there is a need to evaluate some modern interpretations of Nyaya Logic, in terms of formal techniques of modern symbolic or Mathematical Logic. Consequently, such formalization of Nyaya theory of inference leads to certain paradoxes, namely, a paradox of material implication, implication of contradiction $(p \cdot \sim p)$, paradox of confirmation due to equivalence condition in contraposition — $(p \to q) \equiv (\sim q \to \sim p)$ — especially in interpreting scientific laws as true general propositions. I have examined a theory of confirmation in Indian Logic in the light of the contemporary conceptual framework. In order to overcome a difficulty due to equivalence formula, I have made an attempt to reconstruct scientific theory of confirmation in terms of the concepts and techniques in epistemic and modal logics. I think it is possible to formulate a paradox of deduction analogous to that of induction: A formally valid argument, expressed in its conditional form is logically equivalent to its contraposition, but their formal proofs and decision procedure are different. However, in epistemic logical theory of Nyaya system, it is possible to overcome such a paradoxical situation.

Formalization of Theory of inference in Nyaya Logic considers the epistemological significance of four categorical propositions: A, E, I and O propositions as formulated by Aristotle and modified by modern logicians in the tradition of G. Frege. The square of opposition of propositions implies a set of consistent propositions, namely, A and I ; E and O. However, modern version of A proposition as universally quantified conditional form, symbolically $(x)(Fx \to Gx)$, leads to a paradox as the universally quantified conditional proposition turns out to be true even if both Fx and Gx are false. However, A proposition, in its epistemic form is free from a paradox. Epistemic Inferences in Nyaya Logic (Ks: s knows that…, Fm: mountain has fire…):

1. Proposition to be proved: There is fire in the yonder mountain. (Fm)
2. Reason: Because there is smoke in the mountain. $Ks(Sm)$
3. Rule/Law: Wherever there is smoke, there is fire. $Ks((x)(Sx \to Fx))$
4. Application: That mountain has smoke. $Ks(Sm)$
5. Conclusion: Therefore, The yonder mountain has fire. $Ks(Fm)$

Above inference is formally valid with true elements. There are other epistemic valid inferences containing disjunctive, contra-positive and relational propositions. In this system of epistemic logic, there is clear line of demarcation between correct epistemic inferences and incorrect ones. Fallacies in epistemic logic are due to the violation of the rules of epistemic logic. Accordingly, consistency in epistemic logic implies formal validity and truth of the elements.

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6.19 Validity in Paraconsistent Epistemic Logic

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Mediate knowledge has two forms, namely knowledge by deduction and knowledge by induction. This paper is an attempt to investigate the nature of epistemic validity for a given inference with reference to the rules of modern classical formal logic. An inference is epistemically valid if and only if it is formally valid and its elements are true. Usually such inferences are called sound inferences.

However, some formally valid inferences do not yield knowledge if they contain a false premise or a false conclusion. Thus, there is a distinction between sound or epistemically valid and incorrect inferences. Epistemic validity entails reliable or true cognitions in the form of knowledge claims. Accordingly, there is a distinction between epistemic validity and formal validity. A criterion of epistemic validity draws the line of demarcation between epistemically justified true conclusions and formally justified conclusions which do not yield true knowledge claims, as shown below.

Epistemically valid inferences in the propositional logic, namely, Modus Ponens: If a knowing subject s knows that a proposition "$p \to q$" is true and also knows that p is true, then he knows that q is also true. Symbolically, $(K_s(p \to q) \cdot K_s p) \to K_s q$. Similarly in case of predicate theory, some forms yield epistemically valid conclusions: All planets shine by Sun's light and Venus is planet. Therefore, Venus shines by Sun's light. Symbolically, for a knower s the form of argument may be expressed as: $K_s((x)(P_x \to S_x) \cdot P_v) \to K_s(S_v)$.

However, The criterion of epistemic validity restricts the scopes of the rules of quantification, namely, UG,UI, EG and EI in order to get trustworthy information from the inferences.

Paraconsistency in epistemic logic. Considering the frontiers of paraconsistent logic, there are some established approaches which resist the logical status to some of the rules of inference including the inference of explosion. And relevant logic eliminates certain paradoxes. I think Paraconsistent epistemic validity is consistent with the classical definition of knowledge as justified true belief.

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6.20 Non-well-founded Set Theory Recapture

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Naive set theory grants the power of an unrestricted axiom of abstraction. In order to make full use of the resources available to a paraconsistent mathematician, we must understand the full universe of sets given by unrestricted abstraction. This includes the universe of non-well-founded sets. This of recapture of non-well-founded set theory will follow in the, albeit consistent, footsteps of Peter Aczel's work "Non-well-founded Set Theory". The results that can be carried over shall be proved for paraconsistent set theory. This development is of its own merit to paraconsistent set theory. In order to give value and credence to paraconsistent set theory, it must be demonstrated that it can in fact give rise to an interesting and fruitful universe of sets. Exploring paraconsistent non-well-founded set theory adds to this goal. The machinery of non-well-founded sets can also be of use to paraconsistent analysis. I refer to the recent work of Ballard and Hrbáček, "Standard Foundations for Nonstandard Analysis", which demonstrates "constructive use for non-well-foundedness in the foundations of nonstandard analysis". A paraconsistent adaptation of their construction is sought. A construction of the infinitesimal in paraconsistent analysis would provide powerful proof machinery. It is the goal of the second phase to use the non-well-founded set theory to construct the paraconsistent infinitesimal.

6.21 Tableau metatheory for paraconsistent logics defined by possible world's semantics

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The aim of the paper is to demonstrate and prove a tableau metatheorem for paraconsistent logics defined by possible world's semantics. While being effective tableau methods are usually presented in a rather intuitive way and our ambition is to expose the method as rigorously as possible. To this end all notions displayed in the paper are couched in a set theoretical framework, for example: branches are sequences of sets and tableaus are sets of these sequences. Other notions are also defined in a similar, formal way: maximal, open and closed branches, open and closed tableaus.

Thanks to the precision of tableau metatheory we can prove the following theorem: completeness and soundness of tableau systems are immediate consequences of some conditions put upon a class of models **M** and a set of tableau rules **MRT**.

The approach presented in the paper is very general and may be applied to other systems of logic as long as tableau rules are defined in the proposed style. In this paper tableau tools are treated as an entirely syntactical method of checking correctness of arguments. The approach is based on the article [4].

The formal theory presented in the paper offers a simplification of a process of defining all notions and proving particular facts while constructing a modal or paraconsistent tableau system. What is covered by the theory turns out to be all general features of any tableau system determined by possible world semantics. Moreover it allows to define suitably some set of tableau rules in such a way that the sufficient condition for completeness and soundness of the system is satisfaction of the aforementioned conditions. In the standard approach — in contrast to the one presented — it seems to be very difficult to prove general facts about the classes of logics, since we do not have universal and precise notions that are constant and vary only from one set of tableau rules to another.

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6.22 Aristotle's Syllogistic Logic is a Paraconsistent Logic

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Paraconsistent Logic (PL) denies the classical logic (CL) thesis, PC: $p \wedge \neg p \models q$. A strong version is SPC: $\forall\alpha\, \forall\beta\, (\alpha \wedge \neg\alpha \models \beta)$, a contradiction entails any wff. A weak version is WPC: $\forall\alpha\, \exists\beta\, (\alpha \wedge \neg\alpha \models \beta)$, a contradiction entails at least one wff. The denial of PC is PPL: $p \wedge \neg p \not\models q$. A strong version is SPPL: $\forall\alpha\forall\beta\, (\alpha \wedge \neg\alpha \not\models \beta)$; A weak version is WPPL: $\forall\alpha\, \exists\beta\, (\alpha \wedge \neg\alpha \not\models \beta)$. I argue that Aristotle's Syllogistic Logic (ASL) is strong PL (SPL). Contemporary PL (CPL) is weak PL (WPL) as it accepts $\alpha \wedge \neg\alpha \models \alpha$, whereas ASL can deny $\alpha \wedge \neg\alpha \models \alpha$, because $\alpha \wedge \neg\alpha$ is never a line in a proof. Gomes and D'Ottaviano (2010) have established ASL as paraconsistent in a 'broad' as well as a 'strict' sense. CPL is not only 'para', in the sense 'out of' (Beziau 1999, p. 3), coming 'out of' CL, but also coming 'out of' ASL. I take 'classical logic' as it is referred to in the contemporary literature on paraconsistent logic to be modern classical logic, not the classical logic of Aristotle's syllogistics. If CL is an extension of ASL, hence 'out of' ASL, and CPL is an extension of CL, hence 'out of' CL, then, transitively, CPL is an extension of ASL, hence 'out of' ASL. Whereas CPL accepts surfacing of contradictions in proofs, ASL rejects contradictions surfacing in demonstrations. So CPL is 'para' in the sense of 'against' (Ibid., p. 3), being opposed to ASL in this manner. ASL as well as CPL is 'para' in the sense of 'against' CL as they deny SPC. SPC is not found in *Prior Analytics*. WPC may be cited: 'it is possible that opposites may lead to a conclusion, though not always or in every mood' (64^a 15–16). This happens in 6 of the 64 possible syllogisms with contradictories as the premises, where the 'three terms' requirement is circumvented by treating the same term as minor and major. Hence, ASL accepts WPPL. So ASL is at least WPL. Aristotle states 'in the first figure no deduction [...] can be made out of opposed propositions' (63^b 31–2); and 'only that which proceeds through the first figure is irrefutable if true' (70^a 29–30). Axiomatic ASL (Łukasiewicz 1951) has two axioms: 'It is possible to reduce all deductions to the universal deductions in the first figure' (64^a 1). ASL then is WPL if the six syllogisms above can be reduced to the two axioms, if they cannot, then ASL is SPL. Aristotle states two meta-axioms: 'A deduction is a discourse in which, certain things being stated, [ASL.MA1]: *something other than what is stated follows of necessity* from their being so. I mean by the last phrase that [ASL.MA2]: *it follows because of them*, and by this, that no further term is required from without in order to make the consequence necessary'. (24^b 19-22) (my labelling in brackets and my *italics*). Restated: ASL.MA1: The conclusion must be distinct from the premises. ASL.MA2: the conclusion must be contained in the premises. When β is a wff that contains neither α nor $\neg\alpha$, then SPC violates ASL.MA2. Hence, ASL is at least WPL. The contradictions in ASL are between A and O and E and I, hence ASL uses 'negation' as a paraconsistent negation defined by Beziau (1999, p. 11; 2002, pp. 2–3), as 'all humans are primates' may be true in the actual world but 'some humans are not primates' may be true in a possible world. Aristotle makes a distinction between dialectical and demonstrative propositions (24^a 21–24). In a *reductio* argument based on the assumption of a hypothesis, a dialectic proposition, when a contradiction is arrived at then the hypothesis is rejected. This is not SPC as only the negation of the dialectic proposition is entailed. So ASL is WPL. In a demonstration since we begin with demonstrative propositions, there are no hypotheses and every line in the proof is a thesis, then a contradiction will never result as a line of the proof. In this case both SPC and SPPL are vacuously true. Nonetheless, SPPL holds for Aristotle since contradictions are repugnant to the human mind, nothing can be a consequence of a contradiction. SPC leads to a paradox. If anything is implied by a contradiction then CL is violated as the negation of any theorem will be a consequence of a contradiction. CPL resolves the paradox by allowing contradictions to emerge within axiomatic PL, rejecting SPC, while sustaining the rest of CL. ASL resolves the paradox by rejecting SPC and not allowing contradictions to surface in demonstrations. As Beziau (2001) has pointed out, CL's acceptance of PC is based on an equivocation of proof theory with consequence relation. When the consequence relation has primacy over proof theory then PPL has an advantage over PC. A paraconsistent ASL^+ akin to da Costa's C_1^+ can be constructed (da Costa et al). I provide a brief sketch of how this can be done.

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6.23 Report on Kalman Cohen's 1954 Oxford Thesis on Alternative Systems of Logic

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In 1954 Kalman Joseph Cohen submitted to the University of Oxford a BLitt thesis entitled *Alternative Systems of Logic*. The second part of the thesis set out to develop 'a system of logic precisely dual to intuitionistic logic' (p. 188), a project that had been suggested to him by his supervisor Karl Popper. Two features of such a logic that follow immediately from the proposed duality are: (a) the law of explosion is no longer valid in the system, but the law of excluded middle is a theorem; (b) the intuitionistic conditional has to be replaced, or supplemented, by a new connective, called the anticonditional (known also as the operation of difference, subtraction, and co-implication) that is the residual of disjunction in the same way as the conditional is the residual of conjunction.

The thesis does not explicitly provide a system of logic precisely dual to intuitionistic logic in either Heyting's axiomatic formulation or Gentzen's sequent calculus formulation. Instead it provides sequent calculi GK2, GJ2, and GL2 for extensions of classical logic, intuitionistic logic, and its dual, that include rules for both the conditional and the anticonditional. The thesis offers proofs of the normal form theorem, and the decidability, of each of GK2, GJ2, and GL2, proofs that follow the presentation in Chapter XV of Kleene's [1] for Gentzen's classical and intuitionistic sequent calculi. It provides also axiomatic (Hilbert) systems equivalent to each of GK2 and GJ2, and explains briefly why no such axiomatic system is available for GL2. In conclusion some remarks are offered on the interpretation of the anticonditional.

Cohen's thesis was briefly mentioned in Popper's [2], but has otherwise escaped scholarly attention. It is clear, however, that it anticipated more recent work on dual-intuitionist logic. The present paper sets out to evaluate what Cohen achieved, and to what extent his results and their interpretation depend on previous work by Popper. We shall consider also how the results should be viewed in the light of HB (Heyting-Brouwer) logic and other more recent developments.

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6.24 A paraconsistent solution to Kratzer's modal semantics

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In her 1977 article "What 'must' and 'can' must and can mean?" Angelika Kratzer investigates the nature of 'must' and 'can' and tries to find in various meanings of them a smaller but denser meaning which can be applied to all usages of these words. In order to achieve this goal, she puts forward the idea that all modal sentences come with a restriction on 'must' and 'can' so that this restriction relativizes modal phrases. Later, she proposes a semantic approach for modal sentences, which is based on this idea. However, in her semantic approach she encounters several problems. In this presentation, I will embrace the idea that modal sentences come with a restriction and they are constituted by three elements as Kratzer distinguishes. Different than Kratzer, I will elaborate a paraconsistent approach for the construction of the semantics, which is more feasible than her proposal. It solves all problems she raises, and also does not require non-intuitive aspects such as propositions being sets of possible worlds.

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6.25 Hypersequent Calculi for Dual-superintuitionistic Logics and an Extension of the Logic Cube

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Dualization has been used as a useful technique for obtaining paraconsistent logics. Dual intuitionistic logic is a typical case of paraconsistent logic that is motivated by duality consideration on intuitionistic implication or negation (e.g., [3]). However, it appears that there is no unique and canonical way of dualizing a logic. Carnielli et al [2] introduced a bunch of dualized logics in the context of superintuitionistic logics in a way that is semantically well motivated.

In this talk, we take a different way of taking a look at a variety of logics in the class of logics introduced by [2]. We both present a proof-theoretic viewpoint for re-motivating these logics and show some technical results in the proof-theory of these logics. We have at least two different motivations for discussing them here. First, we would like to give a way of systematically presenting a variety of logics with implication and co-implication from a uniform proof-theoretic perspective. This shows that these logics may not only have such semantic motivations as Brunner and Carnielli discuss in [2] but also proof-theoretic motivations if different logics are to be presented from a uniform perspective. Second, it is true that considering co-implication in dual intuitionistic logic (aka “subtraction”) was a well-motivated way of introducing a paraconsistent logic, but, once it is dualized, it is not at least obvious that we have to keep the original constructivist philosophy (one could keep the constructivist philosophical ideas but now they are relatively independent of the logical systems themselves). One may want to introduce logics stronger than dual-intuitionistic logic. Dual-superintuitionistic logics give a way of extending it.

Here is an outline of our talk. First, we introduce the crucial co-implication connective, basically following (but slightly deviating) Sambin et al’s idea of the principle of reflection in [5]. Namely, we start from the following “definitional equation” for co-implication that is common to all of our logics (for the sake of comparison with the ordinary implication, we present both cases here):

1) implication: $\Gamma, A \vdash B$ if and only if $\Gamma \vdash A \to B$;
2) co-implication: $A \vdash \Delta, B$ if and only if $A - B \vdash \Delta$.

Here "Γ, A" and "Δ, B" are multisets of formulas. "$\vdash$" is an appropriate consequence relation that satisfies only reflexivity and transitivity ([1]). As Sambin et al (and other people independently) do, we formulate different logics by modifying structural rules and contextual features for these logics.

Second, we formulate hypersequent calculi for all the dual-superintuitionistic logics which are dual to the superintuitionistic logics whose cut-free hypersequent calculi have already been formulated (including dualized Gödel-Dummett logic and the dualized logic of weak excluded middle). We show cut-elimination for these systems via a proof-theoretic reduction method presented in [4].

Third, after discussing these technical issues, we argue that Sambin et al's logic cube in [5] should be extended via the framework of hypersequent calculi. (Our particular "extension" is not exactly an extension of Sambin et al's logic cube since it contains quantum logic but we are not particularly interested in keeping quantum logic in the scope of our discussion.) We also discuss some issues related to limitations on further extensions of the logic cube via hypersequents. These issues include a comparison of the framework of hypersequent calculi with some other proof-theoretic frameworks such as nested sequents and display calculi and an examination of the view that further extensions may require a proof-theoretical framework more general than hypersequent calculi.

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6.26 A fresh look at the continuum

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In this talk on paraconsistent mathematics, we begin investigations into an entirely new approach to the continuum.

Paraconsistent logics are often derided as being too weak to study mathematics in-depth. In a recent paper, Weber and M^cK-J showed that this is not the case; using a suitable paraconsistent logic, reconstruction of proofs — or creation of new proofs altogether — is possible for most of the desired theorems from classical real analysis. This showed that real analysis is *possible* when using a paraconsistent logic.

In our approach we find that there is more to a continuum than meets the classical eye — or, indeed, any mathematical eye the author is aware of. The approach is top-down, starting with the basic intuition of continuity, and working down to basic properties. Along the way we meet approaches reminiscent of Peirce, Cauchy, Leibniz and, of course, Dedekind. This begins to show that, in the end, real analysis is indeed very *rich* when done paraconsistently.

6.27 On the General Impossibility of a Consistent Theory of Everything

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This is the first part of a two part paper regarding the potential form of a paraconsistent theory of everything and nothing from a metamathematical perspective. This part approaches the question of whether or not a theory of everything can be formulated in a consistent or paraconsistent object logic with a consistent metalanguage. Overall, this part is characterized by using a consistent system of reasoning while assuming in the metalanguage that paraconsistent alternatives in proofs exist though a comprehensive treatment of paraconsistent proofs is beyond the scope of this part of the paper. I conclude that a theory of everything can not be formulated in either a consistent or paraconsistent object logic with a consistent metalanguage and conjecture a potentially novel metamathematical form of recursive paraconsistency.

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6.28 On modal logics defining a Jaśkowski-like discussive logic

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Jaśkowski's discussive logic $\mathbf{D_2}$ was formulated with the help of the modal logic **S5** as follows (see [4, 5]): $A \in \mathbf{D_2}$ iff $\ulcorner \Diamond A^{\bullet} \urcorner \in \mathbf{S5}$, where $(-)^{\bullet}$ is a translation of discussive formulae into the modal language such that:

1. $(a)^{\bullet} = a$, for any propositional letter a,
2. for all discussive formulae A, B:
   $(\neg A)^{\bullet} = \ulcorner \neg A^{\bullet} \urcorner$,
   $(A \vee B)^{\bullet} = \ulcorner A^{\bullet} \vee B^{\bullet} \urcorner$,
   $(A \wedge^{\mathrm{d}} B)^{\bullet} = \ulcorner A^{\bullet} \wedge \Diamond B^{\bullet} \urcorner$,
   $(A \rightarrow^{\mathrm{d}} B)^{\bullet} = \ulcorner \Diamond A^{\bullet} \rightarrow B^{\bullet} \urcorner$,
   $(A \leftrightarrow^{\mathrm{d}} B)^{\bullet} = \ulcorner (\Diamond A^{\bullet} \rightarrow B^{\bullet}) \wedge \Diamond(\Diamond B^{\bullet} \rightarrow A^{\bullet}) \urcorner$.

Thus, the key role in the definition of the logic $\mathbf{D_2}$ is played by the logic **S5**. In a series of papers (see e.g. [3, 7–10]) there are considered other modal logics that are also defining the same logic $\mathbf{D_2}$. Among others, the weakest normal, regular and generally the weakest modal logic defining $\mathbf{D_2}$ were indicated.

In the literature there are considered other translations that are determining other Jaśkowski's like logics. In [1, 6] instead of the original, right, discussive conjunction, the left discussive conjunction is treated as Jaśkowki's one (the other connectives are defined by the same conditions as in the case of the transformation $(-)^\bullet$): $(A \wedge^{\mathrm{d}}_{*} B)^* = \ulcorner \Diamond A^* \wedge B^* \urcorner$. In [2], Ciuciura has shown that the transformation $(-)^*$ yields a logic different from $\mathbf{D_2}$. Ciuciura denotes the obtained logic by '$\mathbf{D_2^*}$'.

Thus, the question arising (which has been stated by João Marcos), what does it change if we consider the weakest in the mentioned classes, modal logics that determine the logic $\mathbf{D_2^*}$. In the paper we will give an answer to this question.

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6.29 Naive set theories based on expansions of BD enriched by classical negation

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There are a number of approaches to Russell's paradox in naive set theories. One of the notable approaches is the dialetheic approach in which concerned contradictions are accommodated as true. Although dialetheism in general does not necessarily exclude the possibility of dealing with classical negation, there are some cases where the presence of classical negation is thought to be not possible. And one of such examples is naive set theory. Note here that there is also a problem of what a classical negation is. Then, the aim of the paper is twofold. First, we review and examine the notion of classical negation in expansions of four-valued logic of Belnap and Dunn (**BD**). Second, we prove that naive set theories based on expansions of **BD** by classical negation are non-trivial, and thus we may claim that it is in fact possible to deal with classical negation even in naive set theory.

In the case of expansions of **BD**, there seems to be at least two kinds of unary operations which are regarded as classical negation in the literature. In this paper, we distinguish them as *exclusion* negation and *boolean* negation. Briefly speaking, the former is a four-valued version of the operation often read as 'It is not true that' in three-valued logic, and the latter is the operation which corresponds to the boolean complementation in algebras. For the purpose of clarifying the differences of these two operations, we make remarks from the semantic perspective, and then outline the proof theory of two expansions of **BD** enriched by exclusion negation and boolean negation. We refer to these expansions as **BD**$*$ and **BD**$+$ respectively.

After these observations, we then turn to consider the naive set theories based on the above two systems. The main background here is the work of Greg Restall on naive set theory based on the Logic of Paradox (**LP**). In order to develop various theories reflecting the stand of dialetheist, an alternative system of logic that replaces classical logic is required. **LP** of Graham Priest has been one of the most popular logics for this purpose. Now, the main result of Restall is that naive set theory based on **LP** is actually non-trivial in the sense that there is a statement that is not provable in the developed theory. There are also some problems related to the theory, noted by Restall as well, but in any case we can talk about naive set theory based on **LP** without the theory being trivial, or logically uninteresting.

Based on these, we show that naive set theory based on **BD**$*$ and **BD**$+$ are also not trivial by following the proof of Restall. The point to be emphasized is the following:

in formulating the axioms of naive set theory, we make use of biconditional. In the case of naive set theory based on **LP**, the material biconditional is defined by paraconsistent negation. On the other hand, in the case of **BD**$*$ and **BD+**, there are two possibilities. However, these two possibilities have not been considered on a par. Indeed, all authors have, at least to the best of my knowledge, taken material biconditional to be definable in terms of classical negation. There is, however, another possibility — viz. to define material biconditional by paraconsistent negation. And it is this latter strategy that leads us to the desired result.

Having the non-trivial proof of several naive set theories, it is then natural to ask the difference among the systems we dealt with. And as an answer to this question, we especially focus on two theories; one based on **LP** enriched by exclusion negation (equivalently, the paraconsistent three-valued extension of **BD**$*$, which is equivalent to **LFI1** and **CLuNs**), and the other based on **BD+**. This is because underlying systems **LFI1** and **BD+** share a lot of interesting properties, but they are different in a significant point, namely the validity of the law of excluded middle. We will thus focus in particular on the role of the law of excluded middle, and observe some differences.

The general lesson that we learn from the result is as follows. The presence of classical negation is not necessarily harmful if we use paraconsistent negation in formulating what are otherwise problematic principles such as naive comprehension. All is not without problems, however. Indeed, we need to examine the problem raised by Restall in the case of the **LP**-based theory, and we also need to give a separate justification for which negation is appropriate in formulating certain principles. For the former, the presence of classical negation might help us, and for the latter, more discussion is required which I take up in the present paper.

6.30 Towards a unified setting for non-monotonicity and paraconsistency

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The notion of *control set* has been introduced and discussed in [3, 5]. A control set is defined as a finite set of contexts $\mathfrak{S} = \{\Lambda_1, \ldots, \Lambda_n\}$. The key idea is that a control set collects all the contexts which are supposed to block a certain derivation. A *controlled sequent* is a sequent decorated with a control set as follows: $\Gamma \vdash_{\mathfrak{S}} \Delta$. The sequent $\Gamma \vdash_{\mathfrak{S}} \Delta$ is said to be *sound* if the context Γ is *compatible* with the control set $\mathfrak{S}$, i.e. for all $\Lambda_i \in \mathfrak{S}$, $\Lambda_i \nsubseteq \Gamma$.

A suitable system of control sets $\mathcal{S}$ can be attached, in principle, to any logical system $\mathscr{L}$. In this way, each application of the rules of $\mathscr{L}$ along derivations has to preserve, besides validity, the soundness of the proved sequent. Informally speaking,

$\mathcal{S}$ assigns to each atom p a control set $\mathcal{S}(p)$ so that the corresponding axiom will be:

$$\frac{}{p \vdash_{\mathcal{S}(p)} p} \text{ ax.}$$

Moreover, $\mathcal{S}$ indicates how to combine and transform control sets along derivations. For instance, it seems quite natural to decorate the LK rule for the right conjunction as follows

$$\frac{\Gamma_1 \vdash_{\mathfrak{S}} A, \Delta_1 \qquad \Gamma_2 \vdash_{\mathfrak{T}} B, \Delta_2}{\Gamma_1, \Gamma_2 \vdash_{\mathfrak{S} \cup \mathfrak{T}} A \wedge B, \Delta_1, \Delta_2} \wedge\text{-right}$$

so that it is soundly applied when the context Γ_1, Γ_2 turns out to be compatible with $\mathfrak{S} \cup \mathfrak{T}$. The idea is that if the contexts in $\mathfrak{S}$ block the derivation of A, Δ_1 from Γ_1 and the contexts in $\mathfrak{T}$ block the derivation of B, Δ_2 from Γ_2, then the contexts in $\mathfrak{S} \cup \mathfrak{T}$ block the derivation of $A \wedge B, \Delta_1, \Delta_2$ from Γ_1, Γ_2.

Now, let $\mathscr{L}^{\mathcal{S}}$ be the controlled calculus obtained from the logic $\mathscr{L}$ be means of a certain system $\mathcal{S}$. A standard sequent $\Gamma \vdash \Delta$ is said to be provable in $\mathscr{L}^{\mathcal{S}}$ if there is a control set $\mathfrak{S}$ such that $\Gamma \vdash_{\mathfrak{S}} \Delta$ is derivable in $\mathscr{L}^{\mathcal{S}}$. It is clear that $\mathscr{L}^{\mathcal{S}} \subseteq \mathscr{L}$. In case we deal with only one control set, the empty one $\emptyset$, we just have $\mathscr{L}^{\mathcal{S}} = \mathscr{L}$.

We will show, at first, how suitable systems of control sets are able to introduce, in a consistent and uniforming way, both non-monotonic and paraconsistent features in almost any logical calculus under consideration. Then, we will focus on paraconsistent controlled versions of LK and provide some possible logic-epistemological relations holding between these calculi and the Logics of Formal Inconsistency [4]. Finally, the debate about the logical hierarchy effectively holding between non-monotonicity and paraconsistency will be taken into consideration [1, 2, 6] and discussed in the light of the control sets-based approach.

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6.31 Internal and External logics of Nelson Models

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Let M be a Nelson model, i.e. an 'information structure' $\langle W, \leq, V^+, V^- \rangle$ where W is a nonempty set of information states partially ordered by the relation $\leq$ and V^+ is a positive, and V^- a negative valuation (both assigning to every atomic formula an upward closed subset of W). With respect to this model the local relations of verification $\left(\Vdash^+\right)$ and falsification $\left(\Vdash^-\right)$ between the information states and formulas of standard propositional language are defined in the usual way. The internal logic $IL(M)$ of the model M is defined as the set of formulas verified at all information states of M.

We will define also an external logic of the model M. For this purpose we will introduce global relations of verification and falsification between upward closed subsets of W and formulas. The global relations correspond to the local relations of the model $M^* = \langle UpW, \supseteq, U^+, U^- \rangle$, where UpW is the set of all upward closed subsets of W, $\supseteq$ is the superset relation and U^+, U^- are defined as follows:

$$U^+(p) = \{X \in UpW \mid X \subseteq V^+(p)\},\ U^-(p) = \{X \in UpW \mid X \subseteq V^-(p)\}.$$

The external logic $EL(M)$ of M is defined as the set of formulas verified at the state W of the model M^*. This logic can be intuitively understood as the logic which concerns the process of localization of an information state (of some agent) in the informational structure M.

We say that a set of formulas Γ is an internal logic if there is a class of Nelson models C such that $\Gamma = \bigcap\{IL(M); M \in C\}$.* It is clear that the paraconsistent Nelson logic $N4$ is the least internal logic determined by the class of all Nelson models.

We will assign to every internal logic L an external logic $E(L)$. Let L be an internal logic and $Mod(L)$ the class of all Nelson models M such that $L \subseteq IL(M)$. Then $E(L) = \bigcap\{EL(M); M \in Mod(L)\}$. The main result of my paper is the following one: For every internal logic L, $E(L)$ is an internal logic identical with the class of formulas

*Notice that internal logics do not have to be closed under universal substitution.

provable in a system of natural deduction containing the rules of Nelson logic:

$(\wedge I)$	$\varphi, \psi \ / \ \varphi \wedge \psi$,	$(\wedge E)$	(i) $\varphi \wedge \psi \ / \ \varphi$, (ii) $\varphi \wedge \psi \ / \ \psi$,
$(\vee I)$	(i) $\varphi \ / \ \varphi \vee \psi$, (ii) $\psi \ / \ \varphi \vee \psi$,	$(\vee E)$	$\varphi \vee \psi, [\varphi : \chi], [\psi : \chi] \ / \ \chi$,
$(\rightarrow I)$	$[\varphi : \psi] \ / \ \varphi \rightarrow \psi$,	$(\rightarrow E)$	$\varphi, \varphi \rightarrow \psi \ / \ \psi$,
$(\neg\wedge I)$	$\neg\varphi \vee \neg\psi \ / \ \neg(\varphi \wedge \psi)$,	$(\neg\wedge E)$	$\neg(\varphi \wedge \psi) \ / \ \neg\varphi \vee \neg\psi$,
$(\neg\vee I)$	$\neg\varphi \wedge \neg\psi \ / \ \neg(\varphi \vee \psi)$,	$(\neg\vee E)$	$\neg(\varphi \vee \psi) \ / \ \neg\varphi \wedge \neg\psi$,
$(\neg\rightarrow I)$	$\varphi \wedge \neg\psi \ / \ \neg(\varphi \rightarrow \psi)$,	$(\neg\rightarrow E)$	$\neg(\varphi \rightarrow \psi) \ / \ \varphi \wedge \neg\psi$,
$(\neg\neg I)$	$\varphi \ / \ \neg\neg\varphi$,	$(\neg\neg E)$	$\neg\neg\varphi \ / \ \varphi$,

plus two more rules in which α ranges over formulas built out of literals only by conjunction and implication (such formulas will be called simple):

$$(L) \quad /\alpha \quad \text{for every simple } \alpha \in L,$$
$$(\rightarrow\vee) \quad \alpha \rightarrow (\varphi \vee \psi) \ / \ (\alpha \rightarrow \varphi) \vee (\alpha \rightarrow \psi).$$

6.32 Eastern Proto-Logics

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Eastern thought has been frequently referred to enhance the philosophical import of paraconsistency, especially with its variant of dialetheism according to which there are true contradictions [4]. Thus, the Indian Jaina logic of saptabhaṅgī and the Chinese Tao have been taken to be philosophical ancestors of paraconsistent logics by dismissing the Principle of Non-Contradiction (a sentence and its negation cannot be true together). In the following, I want to establish the following eight theses (I)–(VIII):

(I) An alternative semantic framework is able to reconstruct and make sense of such alleged “Eastern logics”: a Question-Answer Semantics (thereafter: QAS), including a set of questions-answers and a finite number of ensuing non-Fregean logical values. Thus, the meaning of any object is given by yes-no answers to corresponding questions about its relevant properties [2,5].

(II) These logical values help to show that the saptabhaṅgī (and its dual, viz. the Buddhist Mādhyamaka catuṣkoṭi) is not a many-valued paraconsistent logic [3,5,13] but, rather, a one-valued proto-logic [7,9]: a constructive machinery that serves as a formal theory of judgment, rather than a Tarskian-like theory of consequence [7].

(III) QAS does justice to the central and twofold role of dialectics in Eastern thought, namely: as a linguistic activity of questioning within dialogues, on the one hand; as an ontological process of change in things, on the other hand [9].

(IV) The difference between logical and Hegelian contradiction can be equally rendered within the Boolean algebra and the theory of meaning advocated by QAS: the former is a prohibited relation between a sentence and its negation in a model, whereas the latter is a dynamic process of transformation in things (whether sentential or not). In other words, logical contradiction is a logical property within a ready-made domain, whereas Hegelian contradiction is an ontological property within a continuously expanding domain [1,11].

(V) The Taoist Book of Changes (Yi King) exemplifies such an expansion through its increasing sets of bit strings (guà and bāguà) [10,12].

(VI) A model for Hegelian contradiction follows from (IV) and (V) and results in an expanding domain of logical values, where the initial value is the so-called Absolute (or "the True") that represents a unary string splitting into an indefinite number of other ones by dichotomy. The whole is realized by the Hegelian process of Aufhebung [11]; the latter and the algebraic characterization of the Absolute can also help to explain why the Jains referred to the concept of avaktavyam as an inexpressible truth leading to their attitude of silence [7].

(VII) Such an explanatory model of contradiction assumes a deep redefinition of logical values: a logical value stands for the algebraic characterization of a given individual in a context-sensitive ontology, thus radicalizing the Łukasiewiczian view of logical values as referring now to unique objects (whereas the Fregean "propositions" are special objects to which logical values are to be assigned jointly) [8,9].

(VIII) The core concept of these Eastern theories is not consequence but, rather, opposition: the process of differentiation relies upon the operation of negation and locates the effect of Eastern proto-logics somewhere between formal ontology and formal logic [8,9].

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6.33 Paraconsistent Hermeneutics for Frege and Wittgenstein

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Appealing to methods of hermeneutical analysis for interpreting and understanding logical texts one would indispensably need to attend more closely to some moments, which previously were beyond the scope of philosophical and hermeneutical investigations. One of those turns out to be a non-trivial attempt to resolve the problem of inconsistency of a hermeneutical discourse. Interpretation that intends to furnish understanding in this case appears to be not inconsistent but paraconsistent interpretation.

A peculiarity of the interpretation consists here in that many of familiar reasoning become rejected. For example, reasoning of the kind "if we interpret this statement as A, then if we interpret another statement as B, then we interpret them both as A and B" fails in S. Jaśkowski's discursive logic which is a kind of paraconsistent one (cf. [1, p. 47]). Hence, in paraconsistent discourse the interpretations of hypothetical statements would not been accumulated cumulatively which itself is able to affect the strategy of interpretation.

Notwithstanding a paradoxicality of situation the concept of truth itself also might be a subject of hermeneutical analysis. Actually, G. Frege in his paper from 1918 "The Thought: a Logical Inquiry" [2], in fact, set an example of such an analysis when he envisages a vicious circle in the definition of truth. But does it means that "true" definition of truth is impossible?

He states the scope of application of the notion of truth or falsity — a sense of a sentence. Then he introduces the notion of a thought dispensing with any strict definition. A thought might be the sense of a sentence, which does not imply that the sense of any sentence is a thought; a sentence just express some thought. Whether or not, to be true is a property of thoughts and not the things. Frege proposes to single out an area for thoughts: some particular third realm. Elements of this realm corresponding both ideas and things being neither perceived by the senses nor belonging to the contents of consciousness of some bearer. And what is of most importance, thoughts are tightly connected with truth. As a consequence, truth would not come into world the other way than in the moment of its discovering. But it is precisely the way a thought act while being apprehended and taken to be true: Thus, truth is now depends on thoughts, they determine its properties and it seems that the vicious circle is broken.

Nevertheless, we need to make certain whether our discourse of interpretation would be inconsistent for the definition of the discursive systems of Jaśkowski's type intends mutual incoordination of pairs of particular contentions. But properly speaking this role is playing by the initial contentions fixing a circle in reasoning, that is contentions of the type "in any definition of the truth indication of some characteristics is included" and "it is indispensable in any particular case to decide whether it is true that justifying characteristics are available".

L. Wittgenstein criticized the conception of thought proposed by G. Frege in connection with Fregean assertion sign, that is, judgment stroke ('Urteilstrich'). The distinction between the positions of Frege and Wittgenstein consists in that for Frege thought indicates a state of affairs, refers to it but says nothing about it (he considers thoughts as complex names) while Wittgenstein thinks that giving names a significant configuration produces something generically different from a name: a fact. However, for the hermeneutic philosopher the most interesting is not a question of either Frege or Wittgenstein was right but rather a fruitfulness of conceptions and methods they used for resolving the task set.

If we consider the well-known problem of the interrelationship between the language and the world in Wittgenstein's Tractatus, then from the point of view of hermeneutics it takes shape of the familiar circle structure since it concerns of the part-whole mutual relations. The language is understood through the world and the other way round which involves that one need to have in his disposal some pre-structure of understanding, some discourse which allows to interpret Wittgenstein's position and unlock the circle.

However, for Wittgenstein the language is a mirror of the world and there is no gap between them allowing introducing some new discourse. For Wittgenstein the proposition indicates the fact and there is nothing more to say since the world consists of facts. But for Frege the proposition needs yet an assertion of its truth, the proposition has to be interwoven into world structure. And this function is accomplished by the thoughts which become the medium between the language and the world. It is not of an accident that Frege introduced the assertion sign in order to account for suppositions: following the course of our consideration the content of proposition becomes as it would be prefixed with the familiar reservation "for a certain admissible meaning of the contention used".

As to Wittgenstein then an opportunity to unlock the emerging hermeneutic circle would be closely connected with the doctrine of "showing" which central theses in Tractatus look as follows. A transcendental character of logic by Wittgenstein's opinion is manifested in that the propositions of logic show something that pervades everything sayable and therefore is itself unsayable.

This unsayable of which the most significant is the 'logic of the world' or the 'logic of facts' is capable in case of Wittgenstein plays the same role as the world of thoughts plays in case of Frege. For it might be said now that in the world-language problem the third part emerges — the world of unsayable. An unsayable is not beyond the world (because it pervades the world) but the world of unsayable does not coincide with the actual world since an attempt to say what it is the 'logic of facts' that is reproduced by sentences leads to stammering.

Now the language becomes "setting apart" the world being defined relatively the world of unsayable too. What does the phenomenon of such estranging means from the point of view of hermeneutics? Let us recall Frege's statement: "Without wishing to give a definition, I call a thought something for which the question of truth arises" which was used in order to introduce the notion of thought into discussion. By parity of reasoning Wittgenstein's step should be characterized as "Without wishing to give a definition, I shall say that there is a something which can be just shown and which pervades everything sayable and therefore is itself unsayable" (properly speaking the lack of definition is precisely the consequence of unsayability). But the last might again be reformulated in Jaśkowski's style as "For a certain admissible meaning of the contention used there is a something which can be just shown and which pervades everything sayable and therefore is itself unsayable". And a characteristics of the connection between tautologies and unsayable might be respectively rewritten as "if it is interpreted as (stated that) there is a something which can be just shown and which pervades everything sayable and therefore is itself unsayable then the connections between the tautologies, or senseless propositions of logic, and the unsayable things that are 'shown', is that the tautologies show the 'logic of the world' ".

The further course of reasoning as in Frege's case is obvious: one need to check the discourse of interpretation for the presence of inconsistencies according to the definition of Jaśkowski's type discursive systems because the definition intends mutual incoordination of pairs of particular contentions. But in given case we deal with the "classical" hermeneutical part-whole contradiction: the world is determined by means of the language which is the part of the world, while the language is determined by the world comprising everything including the language.

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6.34 Iterated preferential models as a strategy to make many-valued paraconsistent logics non-monotonic

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In his 1980's paper John McCarthy used the well known example about missionaries and cannibals to introduce a new concept of non-monotonic reasoning, namely circumscription. He formalized the idea that in such examples only those pieces of information are used which are explicitly stated, i.e. that a boat might be leaky or some of the cannibals could easily swim to the other bank of the river is not considered. Model-theoretical circumscription can be obtained by minimizing the extensions of some predicates with preferential models. But since circumscription is based on classical first order logic it leads to explosion when faced with inconsistent information. In McCarthy's example, this could mean one of the cannibals carries a talisman which is, let's say, yellow and not yellow at the same time. Still circumscription is widely used in computer science as one of the most prominent non-monotonic approaches. If one wants to overcome classical logic's shortcomings with inconsistent reasoning but keep non-monotonicity one could use as basic logic not FOL but a paraconsistent logic, e.g. LP, adapt the definitions for circumscription and obtain a paraconsistent version of circumscription. This was done in Zuoquan Lin's paper from 1996. There he proposed a paraconsistent version of circumscription by combining the minimization of inconsistencies of LPm and the minimization of extensions in circumscription. However, his definition turns out to be problematic, since it can not properly handle inconsistent models.

In this presentation, I want to introduce a new approach, which I call iterated preferential models. The basic idea is that one first minimizes inconsistencies like for example in LPm and then minimizes in the resulting models the extensions of predicates like in circumscription. As basic logic I take instead of FOL at first LPm to show how this approach works. Then I want to prove that every consequence that is valid in circumscription based on FOL is valid in a paraconsistent version of circumscription based on LPm. Further results will include LFI1 and Belnap's FOUR for which some preferential models were introduced in 2003 by O. Arieli and M. Denecker. Finally we come back to the slightly extended example from McCarthy:

Three missionaries and three cannibals who carry a yellow and not yellow talisman come to a river. A rowboat that seats two is available. If the cannibals ever outnumber the missionaries on either bank of the river, the missionaries will be eaten. How shall they cross the river?

In my approach, as I will show, it then easily follows that all missionaries and all cannibals including all of their inconsistent talismans reach safely the other bank of the river. No one will be eaten and pigs still can't fly.

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6.35 Adaptive Logics and Selection Functions: A Generic Format

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Adaptive logics (ALs) are a branch of non-monotonic logics, originally developed in the context of paraconsistent reasoning. Nowadays, there are also ALs for dealing with (prioritized) conflicting norms, ALs that capture reasoning with various types of assumptions, ALs for enumerative induction, etc. All of these have been studied from a more general perspective in terms of a *standard format* [1].

Every AL in standard format is defined by a triple: (i) a compact Tarski-logic **L**, which is often called the *lower limit logic* of the AL; (ii) a set of abnormalities Ω, which are formulas in the object language of **L**, and finally (iii) a *strategy*, which is either *reliability* or *minimal abnormality*. **L** represents the monotonic core of a given AL. Abnormalities are assumed to be false, unless the premises prevent so. Finally, the strategy specifies a way to react (in a uniform way) to cases in which the premises entail a disjunction of abnormalities, but none of its disjuncts.

The aim of this talk is twofold. First, we want to show how the third element in the standard format, viz. the adaptive strategy, can be generalized. This move is inspired by similar work in the field of non-monotonic logic. In particular, it draws on the notion of a selection function as studied in [2].

Second, we show how the generalized format enables us to model various types of reasoning with conflicting information in a natural way. In particular, we consider contexts in which such information is offered by various experts, and we try to merge it in a non-trivial way. In addition, we show that various previously developed ALs can be expressed in the new format, including lexicographic ALs [5, 6], ALs that use non-standard strategies such as (variants of) *counting* [4] or *normal selections* [3], and the prioritized ALs from [7].

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6.36 The Logic LS_3 and its Comparison with other Three-Valued Paraconsistent Logics

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Paraconsistent logic and three-valued semantics: The term *Paraconsistent* was first used by the Peruvian philosopher Francisco Miró Quesada in the Third Latin America Conference on Mathematical Logic in 1976. A logic is called paraconsistent if there are formulas ϕ and ψ such that $\{\phi, \neg\phi\} \nvdash \psi$. Besides other semantics, the three-valued semantics of (paraconsistent) logics have always received special attentions from logicians like J. Łukasiewicz, S.C. Kleene and others and paraconsistentists like S. Jaśkowski, N.C.A. da Costa, G. Priest, R. Brady, C. Mortensen, D'Ottaviano, W.A. Carnielli, João Marcos etc. Parainconsistency axioms have been introduced in [9] in a way similar to classical two-valued logic.

Introduction of the three-valued matrix PS_3: Here we shall introduce a new three-valued matrix, $PS_3 := \langle \{1, \frac{1}{2}, 0\}, \wedge, \vee, \Rightarrow, ^* \rangle$ where $\langle \{1, \frac{1}{2}, 0\}, \wedge, \vee \rangle$ is a distributive lattice and the *designated set* of PS_3 has been fixed as, $\{1, \frac{1}{2}\}$. From the truth tables of PS_3 it can easily be verified that $(\frac{1}{2}) \wedge (\frac{1}{2})^* \Rightarrow 0 = 0$ and hence PS_3 might be a three-valued semantics of some paraconsistent logic.

Proof theory for PS_3: The main part of this work is to develop a propositional logic LPS_3 so that PS_3 becomes the three-valued semantics of it. We have proved that LPS_3 is sound and complete with respect to PS_3. It will then be discussed how does LPS_3 satisfy Jaśkowski's criterion (cf. [2]) of being a paraconsistent logic.

Comparison with other existing three-valued paraconsistent logics: A comparison between LPS_3 and some other paraconsistent logics having three-valued semantics, such as LP (*Priest's Logic of Paradox*) [4], LFI1 (*Logic of Formal Inconsistency 1*) and LFI2 (*Logic of Formal Inconsistency 2*) [10], J_3 (*D'Ottaviano's logic*) [5], RM_3 [8], P1 (*Sette's three-valued logic*) [1], $C_{0,2}$ (*Mortensen's paraconsistent logic*) [3] will be made. Particularly PS_3 has close connections with the three-valued models of the paraconsistent logics P1 (or $C_{0,1}$) and $C_{0,2}$. It is worthwhile to show, how do these logics differ pair wise. It is proved that LPS_3 is *maximal* relative to the *classical propositional logic*. It is also proved that LPS_3 is *maximally paraconsistent in the strong sense*, defined in [7].

Paraconsistent set theory: The motivation of finding the algebra PS_3 is to build a model of some paraconsistent set theory. The paraconsistent logic LPS_3 can be used in some algebra-valued set theory construction similar to the Boolean-valued construction (cf. [6]) to obtain a model of a (weak) paraconsistent set theory.

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6.37 Two Paraconsistent Semantics for Pavelka's Fuzzy Logic

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This work falls in the realm of many-valued paraconsistent logics. Classical logic and most standard non-classical logics are *explosive*; from contradictory premises anything can be inferred (*ex contradictione quodlibet*). Paraconsistent logic challenges this stand point. A logical consequence relation is said to be *paraconsistent* if it is not explosive; even if we are in certain circumstances where the available information is inconsistent, the inference relation does not explode into triviality. Thus, paraconsistent logic accommodates inconsistency in a sensible manner that treats inconsistent information as informative.

In Belnap's paraconsistent logic [1], four possible values associated with a formula α are *true*, *false*, *contradictory* and *unknown*: if there is evidence for α and no evidence against α then α obtains the value *true* and if there is no evidence for α and evidence against α then α obtains the value *false*. A value *contradictory* corresponds to a situation where there is simultaneously evidence for α and against α and, finally, α is labeled by value *unknown* if there is no evidence for α nor evidence against α. A fundamental feature of paraconsistent logic is that truth and falsehood are not each others complements.

In [3] it was shown how Belnap's ideas can be extended to Pavelka's fuzzy logic [4] framework; starting from an evidence pair $\langle a, b\rangle$ on the *real unit quare* and associated with a propositional statement α, we can construct evidence matrices expressed in terms of four values t, f, k, u that respectively represent the logical valuations *true*, *false*, *contradictory* and *unknown* regarding the statement α. The components of the evidence pair $\langle a, b\rangle$ are to be understood as evidence for and against α, respectively. Moreover, the set of all evidence matrices can be equipped with a complete MV-algebra structure. Thus, the set of evidence matrices can play the role of truth-values of Pavelka's fuzzy logic, a rich and applicable mathematical foundation for fuzzy reasoning, and in such a way that the obtained new logic is paraconsistent.

In [2] it was proved that a similar result can be also obtained when the evidence pair $\langle a, b\rangle$ is given on the *real unit triangle*. Since the real unit triangle does not admit a natural MV-structure, we introduced some mathematical results to show how this shortcoming can be overcome, and another complete MV-algebra structure in the corresponding set of evidence matrices is obtained.

In this paper we recall these two approaches, show their differences and similarities and possible real life applications.

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6.38 A comparative study of selected filtered paraconsistent logics

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Interesting paraconsistent logics are obtained by filtration if its consequence operation is the classical consequence restricted by a (hopefully interesting and well-motivated) additional condition on the relation between the premises and the conclusion(s). We will study the following consequence relations (their definitions are preceded by a few preliminary ones). In what follows, $\models$ without subscript stands for classical consequence relation.

Definition 1. Logical cover and its width. $\Im(\Delta)$ is a *logical cover* of a set of formulas Δ iff it is an indexed set starting with $\emptyset$ such that every element Δ_i of $\Im(\Delta)$ is consistent and Δ is a subset of the union of (classical) deductive closures of all elements of the cover: $\Delta \subseteq \bigcup_{i \in I} \mathbb{C}_{CL}(\Delta_i)$. If $\Im(\Delta)$ is of cardinality n, then $n-1$ is its *width* $w(\Im(\Delta))$. ■

Often, sets have more than one logical cover. In such a case, we'll use $\Im$ with subscripts. Not all sets have logical covers. If Δ contains an absurd formula $\alpha\wedge\neg\alpha = \phi$, at least one of the elements of Δ (namely ϕ) cannot be an element of a union of consequence sets of consistent sets.

Definition 2. Level of a set of formulas. The level of Δ is the minimal width of a logical cover of Δ.

$$\ell(\Delta) = min\{w(\Im_k(\Delta)) \mid 1 \leq k \leq c, c \text{ is the number of logical covers of } \Delta$$

If Δ has no cover $\ell(\Delta) = \infty$. If $w(\Im_i(\Delta)) = \ell(\Delta)$ we'll say that $\Im_i(\Delta)$ is a minimal logical cover of Δ. ■

While there can be many different logical covers of a set, every set has a unique level (if the set contains an absurd formula, its level is ∞).

Definition 3. Weak Rescher-Manor consequence. A formula α is an RM_W-consequence of Γ iff it is a classical consequence of at least one mcs of Γ. In other words: $\Gamma \mathrel{|\!\!\!\!\equiv_{RM_W}} \alpha$ iff there exists a set $\Gamma_j \subseteq \Gamma$ such that Γ_j is an mcs of Γ and $\Gamma_j \models \alpha$. ■

Definition 4. Strong Rescher-Manor consequence relation. A formula α is an RM_S-consequence of Γ provided that it is a classical consequence of all the mcss of Γ. In other words, $\Gamma \mathrel{|\!\!\!\!\equiv_{RM_S}} \alpha$ iff for all $\Gamma_j \subseteq \Gamma$, if Γ_j is an mcs of Γ, then $\Gamma_j \models \alpha$. ■

Definition 5. Free consequence relation. A formula α is an F-consequence of Γ iff it is a classical consequence of the intersection of all mcss of Γ. In other words, if $\Gamma_1, \Gamma_2, \ldots, \Gamma_j$ are all mcs of Γ, $\Gamma \mathrel{|\!\!\!\!\equiv_{F}} \alpha$ iff $\Gamma_1 \cap \Gamma_2 \cap \cdots \cap \Gamma_j \models \alpha$. ■

Definition 6. Argued consequence relation. A formula α is said to be an A-consequence of Γ iff there is an mcs of Γ, Γ_j, such that $\Gamma_j \models \alpha$ and there is no mcs Γ_k of Γ such that $\Gamma_k \models \neg\alpha$. ■

Definition 7. Cardinality based consequence relation. A formula α is said to be a CB-consequence of Γ iff α is a classical consequence of every mcs Γ_j of Γ with highest cardinality. ■

Definition 8. Level-forcing consequence relation. A formula α is an LF-consequence of Γ iff every minimal logical cover of Γ contains at least one set which classically entails α. If $\ell(\Gamma) = \infty$, then the only LF-consequences of Γ are the consequences of the empty set (which by definition is built into any logical cover), that is, classical tautologies. ■

Our goal is to compare strength of these paraconsistent systems obtained through filtration and to further investigate their structural properties. While some the facts investigated are already known, our task is to add new results, to provide didactically useful examples regarding philosophical motives for choosing either of them and to streamline and systematize results already available.

6.39 Paraconsistency and Impossible Worlds

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Genuine modal realism is traditionally deemed as not able to include impossibilia into its ontology. It was showed that postulating such an ontology would collapse into plain contradiction and, consequently, to an utter nonsense. The aim of the paper is to show that it does not have to be the case. My investigation of the matters proceeds from a simple assumption that concrete impossible individuals exist. Provided that reader accepts the assumption, the task of the author will be to deal with (at least) three important issues. Firstly, it will have to be shown that genuine modal realism is a respectable metaphysical position and its extension by concrete impossibilia is worth of philosophers' efforts. Secondly, the way of how such an extension should go in order to sustain modal realism's main theoretical virtues will be pursued. Finally, it will be argued that modifying one's account of logical consequence in order to accommodate impossibilia is a mistake. For, as Daniel Nolan puts it, if there is an impossible situation for every way we say that things cannot be, there will be impossible situations where the principles of any subclassical logics fail. To meet the above challenges is the main author's goal. Namely, it will be argued that it is possible to deal with these challenges while staying squarely within the boundaries of modal realism's fundamental framework.

6.40 Reltoses for Relevant Logics

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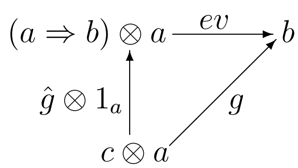
Early in [1] categorical semantics for relevant logics was proposed which was based on reformulation of relevant algebra as a preorder category endowed with some functors mirroring the properties of relevant negation and entailment — so-called RN-category. A modification of RN-categories for conveying properties of relevant algebras allows introduce another version of categorical semantics for relevance being, in essence, a non-classical modification of a topos. This categorical construction would be called a *reltos* pursuing the analogy with the "classical" topos.

Definition 1. An *R-reltos* $\mathcal{C}$ is a (groupoidal) category endowed with a covariant bifunctor $\otimes : \mathcal{C} \times \mathcal{C} \to \mathcal{C}$ such that:

(i) $\mathcal{C}$ has finite products $\langle -, - \rangle$, coproducts $[-, -]$ and $\mathcal{C}$ is distributive relatively to those, i.e. $\langle [a, b], [a, c] \rangle \cong [a, \langle b, c \rangle]$ for any objects a, b, c in $\mathcal{C}$;

(ii) for any objects a, b, c in $\mathcal{C}$ there are the following natural isomorphisms:
$a \otimes [b, c] \cong [a \otimes b, a \otimes c]$,
$[b, c] \otimes a \cong [b \otimes a, c \otimes a]$,
i.e. bifunctor preserves coproducts;

(iii) $\mathcal{C}$ allows exponentiation relative to $\otimes$, i.e. the following diagram commutes:



where $\Rightarrow$ is an exponential;

(iv) the following functorial equations are satisfied:
(a) $(g_1 f_1) \otimes (g_2 f_2) = (g_1 \otimes g_2)(f_1 \otimes f_2)$;
(b) $1_A \otimes 1_B = 1_{A \otimes B}$, for any objects a, b, c in $\mathcal{C}$;
(c) $a \otimes (b \otimes c) \cong (a \otimes b) \otimes c$.

Definition 2. An R-reltos $\mathcal{C}$ is *monoidal* if:

(i) $\mathcal{C}$ has an object e such that $e \otimes a \cong a$ and there is an arrow $a \to e$ in $\mathcal{C}$ for all a in $\mathcal{C}$;

(ii) for any objects a, b, c of $\mathcal{C}$, $a \otimes (b \otimes c) \cong (a \otimes b) \otimes c$.

Definition 3. A monoidal R-reltos is *symmetric monoidal* if for any objects a, b in $\mathcal{C}$ there is an arrow $a \otimes b \to b \otimes a$.

Definition 4. A symmetric monoidal R-reltos is *relevant* if for any object a in $\mathcal{C}$ there is an arrow $a \to a \otimes a$.

Definition 5. An *RN-reltos* $\mathcal{C}$ is an R-reltos together with a contravariant functor $\mathcal{N} \colon \mathcal{C} \to \mathcal{C}$ such that

(i) $\mathcal{N}^2 a \cong a$, for any a in $\mathcal{C}$;

(ii) for any arrow $a \otimes b \to c$, there is an arrow $a \otimes \mathcal{N} c \to \mathcal{N} b$ in $\mathcal{C}$.

It is straightforward to check that any reltos has the following properties:

- $\mathcal{N}\langle a, b \rangle \cong [\mathcal{N} a, \mathcal{N} b]$;
- $\mathcal{N}[a, b] \cong \langle \mathcal{N} a, \mathcal{N} b \rangle$;
- an RN-reltos $\mathcal{C}$ (based on symmetric monoidal R-reltos) $\mathcal{N}(a \otimes \mathcal{N} b)$ is an exponential unique up to isomorphism.

A reltos $\mathcal{C}$ is an RN-reltos having a subobject classifier i.e. a $\mathcal{C}$-object Ω together with an arrow $true : e \to \Omega$ that satisfies the following axiom:
for each monic $f\colon a \rightarrowtail d$ there is one and only one arrow $\chi_f\colon d \to \Omega$ such that

$$\begin{array}{ccc} a & \overset{f}{\rightarrowtail} & d \\ {\scriptstyle !}\downarrow & & \downarrow{\scriptstyle \chi_f} \\ e & \underset{true}{\longrightarrow} & \Omega \end{array}$$

Since any reltos is a category equipped with products, coproducts, bifunctor and functor then truth-arrows in a reltos $\mathbf{C}$ with the subobject classifier
$true\colon e \to \Omega$ would be defined as follows:

- $false\colon e \to \Omega$ is a character of the arrow $0 \to e$;
- $\neg\colon \Omega \to \Omega$ will be the unique arrow for which the diagram

$$\begin{array}{ccc} e & \overset{false}{\rightarrowtail} & \Omega \\ \downarrow & & \downarrow{\scriptstyle \neg} \\ e & \underset{true}{\longrightarrow} & \Omega \end{array}$$

  will be the pullback in $\mathbf{C}$. Thus, $\neg = \chi_{false}$;
- $\cap\colon \Omega \times \Omega \to \Omega$ is a character of the product of arrows $\langle true, true\rangle : e \to \Omega \times \Omega$ in a reltos $\mathbf{C}$;
- $\cup\colon \Omega \times \Omega \to \Omega$ is by definition a character of the image of the $\mathbf{C}-$arrow $[\langle true_\Omega, 1_\Omega\rangle, \langle 1_\Omega, true_\Omega\rangle] : \Omega + \Omega \to \Omega \times \Omega$;
- $\odot\colon \Omega \times \Omega \to \Omega$ is a character of the $\mathbf{C}-$arrow $\Omega \otimes \Omega \to \Omega \times \Omega$ which we obtain from $a \otimes [b, c] \cong [a \otimes b, a \otimes c]$;
- $\supset\colon \Omega \times \Omega \to \Omega$ would be defined as a character of an equalizer of the pair $\Omega \times \Omega \overset{\cap}{\underset{pr_1}{\rightrightarrows}} \Omega$.

Theorem. If $\vdash_R \alpha$, then, for any reltos C, we have $C \models \alpha$.

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6.41 Paraconsistent and classical negation in the context of relevant implication

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In this talk I propose a new way of looking at paraconsistent and classical negation in the relevant logic **R**. I will argue that what I will call the received view (based on what is presented in [1] and [2] is problematic and that an alternative view is at least worth considering.

Relevant logic is the endeavour to save classical logic (henceforth **CL**) from the paradoxes of material implication. The pre-theoretic idea of an implication requires a connection of some kind between antecedent and consequent, i.e. A pre-theoretically implies B iff reasons to accept A give us reasons to accept B. Famously, **CL**'s implication, the material implication, does not require such a connection at all. This leads to all kinds of completely counterintuitive **CL**-consequences with respect to this implication. Among the many strange consequences involved, we have the paradox of irrelevance $B \vdash_{\mathbf{CL}} A \supset B$, i.e. if B is true, A implies B, even if A is completely unrelated to B, and $\vdash_{\mathbf{CL}} (A \supset B) \vee (B \supset C)$. Independent of the truth of A, B, and C and of the question whether they are in any sense related, either A implies B or B implies C. No sensible human would agree with this kind of implication in natural language reasoning. Relevant logics in general, and **R** in particular, succeed in avoiding these 'paradoxes of material implication'.

However, relevant logics do not only avoid the problems of material implication, they also affect the meaning of another connective: negation. One can neither call the negation of **R** classical nor can one refer to it as fully paraconsistent. It is a borderline case. In the Routley-Meyer semantics for **R** the negation is classical in the possible worlds (among which is the actual world) and paraconsistent in the impossible worlds.

So, we do not have a classical negation in relevant logic **R**, nor do we have a real paraconsistent negation in it. The question is natural: can we add both such negations to relevant logic? In their 1973 and 1974 papers Meyer and Routley have presented a proposal for adding classical negation to relevant logic. I argue that this proposal is not entirely successful. The classical negation in these papers is indeed classical, but it renders their implication irrelevant. Let $\neg$ stand for classical negation, $\sim$ will stand for paraconsistent negation. If one takes the relevance seriously, $(A \wedge \neg A) \rightarrow B$ should not be a theorem whenever $\rightarrow$ is meant to be a relevant implication. There is no relevant connection between the antecedent $(A \wedge \neg A)$ and the consequent (B). Of course a classical contradiction can never be true, and therefore it classically entails whatever sentence. But this does not yet mean that it relevantly implies B. It is not because a classical contradiction can never be true, that therefore reasons to accept it would give you reasons to accept any sentence. This goes entirely against the ideas behind relevance.

What can a classical negation be in the context of a theory of relevant implication (if it is not what Meyer and Routley have proposed)? How can we distinguish it from a paraconsistent negation if $(A \wedge \neg A) \rightarrow B$ should not come out as always true (for either of both negations)? I argue that the difference lies in two aspects:

(1) The law of disjunctive syllogism should be valid for classical negation and so $(A \wedge (\neg A \vee B)) \rightarrow B$ should be a theorem. This law should not be valid for paraconsistent negation and so $(A \wedge ({\sim} A \vee B)) \rightarrow B$ should not be a theorem. The same holds for variants like $(A \vee (B \wedge \neg B)) \rightarrow A$. Remark that the standard negation of **R** is paraconsistent with respect to this criterion.

(2) Given some consequence relation $\vdash$ defined on the basis of the theorems of our relevant logic, it should be the case that $A, \neg A \vdash B$ and $A, {\sim} A \nvdash B$.

Although this last difference seems trivial given usual approaches to paraconsistency, it is not obvious for the relevant logic context. Relevant logicians did not care too much about defining consequence relations, and it might therefore seem strange to make distinctions between paraconsistency and classicality of negation in relevant logic on the basis of a consequence relation. Nevertheless paraconsistency is a phenomenon that can only properly be studied at the level of consequence (e.g. Priest's logic **LP** is prototypically paraconsistent but shares all theorems with classical logic and differs only from classical logic at the level of consequence). Whether the standard negation of **R** is paraconsistent with respect to this criterion, depends on how consequence is defined.*

So if one wants a logic containing a relevant implication, a proper paraconsistent negation and a proper classical negation, the logic should at least satisfy:

(i) $\vdash (A \wedge (\neg A \vee B)) \rightarrow B$;

(ii) $\nvdash (A \wedge ({\sim} A \vee B)) \rightarrow B$;

(iii) $A, \neg A \vdash B$;

(iv) $A, {\sim} A \nvdash B$;

(v) $\nvdash (A \wedge \neg A) \rightarrow B$.

And it should of course not validate any of the familiar paradoxes of material implication.

In the final part of my talk I will present a way to add a classical and a paraconsistent negation to **R** which satisfies all these criteria. The semantics introduces an infinity of truth values $(-\omega, \ldots -2, -1, 0, 1, 2, \ldots, \omega)$ to all worlds (possible and impossible) of the Routley-Meyer Semantics. It has the remarkable feature that the relevant implication becomes non-transitive (because $(A \wedge \neg A) \rightarrow ((A \vee B) \wedge \neg A)$ and $((A \vee B) \wedge \neg A) \rightarrow B)$, but I argue that this is exactly as it should be if one wants both a fully classical negation AND a relevant implication.

*Loosely put: it is classical if one defines semantic consequence based on truth in possible worlds (A is a consequence of Γ iff A is true in all possible worlds in which Γ is true) and paraconsistent when it is based on truth also in impossible worlds.

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6.42 Finitism in Paraconsistent Mathematics

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Take **finitism** to be the thesis that there are only finitely many objects, and **paraconsistent mathematics** to be ordinary mathematical practice embedded in a non-explosive consequence relation. Several considerations [3,4] point a paraconsistent mathematician toward a strict finitistic perspective — that one can do all of mathematics with only finitely many objects. For a start, any characterization of infinity seems to require appeal to negation — and if the negation is paraconsistent, then the 'infinity' so characterized is too. More generally, in a paraconsistent setting one can often identify distinct objects together, without losing any truths [1]; so it is fairly easy to show that foundational theories like arithmetic have *finite models* [2], while set theory has a strong *quantifier elimination theorem* [5]. The main result I will discuss is partial answers and modifications to the question:

Under what conditions does a paraconsistent theory have a finite model?

Technicalities in hand, we can consider degrees of finitism, from weak to strong. I'll suggest that a paraconsistent mathematician can be a *radical* finitist, since nothing — including full use of the infinite! — is lost by doing so.

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6.43 Intensionality preserving negation

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Verbs of propositional attitude are more strongly intensional than the classical operator of necessity. On way to show this difference is to point out that intentionality of verbs of propositional attitudes can be tested by a pair of materially equivalent sentences such that both members of the pair are contingent sentences. This is not the case with the operator of necessity: we can show that this operator is intensional only using a pair of equivalent sentences such that one of the sentences is contingent and the other one non-contingent (necessarily true). This observation leads to the following definition of *normal intentionality*:

Definition 1. A sentential operator O (an expression of the category S/S) is *normally intensional* iff for every contingent sentence P and every possible world w, if $O(P)$ is true in w , then there exists a contingent sentence P' such that P and P' have the same truth-value in w and $O(P')$ is false in w.

The second step is to define a *normal negation n-O* of a normally intensional operator O. Roughly speaking, we want the operator n of category (S/S)/(S/S) when applied to a normally intensional sentential operator give as result a normally intensional sentential operator:

Definition 2. For any P, any world w and any operator O, $n\text{-}O(P)$ in w is:
(i) $v(n\text{-}O(P)) = v(\neg O(P))$ in w,
(ii) there exists a sentence P' which has the same truth value as P in w and such that $\neg O(P)$ and $O(P)'$ have the same truth value in w.

We can now prove the main result of this note:

Proposition 1. If O is a normally intensional operator such that for any P, $O(P)$ entails P, then $n\text{-}O(P)$ also entails P.

Proof. Suppose *a contrario* that there exists a world w such that $n\text{-}O(P)$ is true in w and P is false. This means, given D2, that there exists P' with the same truth value as P in w and such that $O(P')$ is true in w. But this is impossible since $O(P')$ entails P'. □

The following definitions precise the notion of the "strength of intensionality":

Definition 3. The pair of sentences $\{P, P'\}$ is a detector of intensionality of the operator O (in the world w) iff P and P' have the same truth value in w but $O(P)$ and $O(P')$ have different truth values in w.

Definition 4. The sentential operator O is intensionally stronger than O' iff the set of detectors of intensionality of O' is strictly included in the set of detectors of intensionality of O (in a given world w).

Given these definitions the following can be proven:

Proposition 2. If O is intensionally stronger than O' and, for any P, $O(P)$ entails $O'(P)$, then n-$O(P)$ entails $O'(P)$.

Proposition 3. If O and O' have the same intensional strength and $O(P)$ entails $O(P')$, then n-$O(P)$ entails n-$O'(P)$.

The above propositions can be illustrated by factive (emotive vs non-emotive) (intensional) verbs (*know* vs *regret*) and by neg-transportable intensional verbs (*believe*).

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